Day 163 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arc length and sector area (All boards)

A sector of a circle has arc length 20 cm and area 80 cm^2.
Find the radius of the circle and the angle of the sector, in radians.

Area = $\frac{1}{2}rs$: $80 = \frac{1}{2} \times r \times 20$

$r = \frac{2 \times 80}{20} = 8$ cm

$\theta = \frac{s}{r} = \frac{20}{8} = 2.5$ rad

Answer: r = 8 cm, θ = 2.5 rad

2 Normal approximation to the Poisson (CIE)

X ~ Po(25). Use a suitable approximation to find P(X ≥ 26), to 3 significant figures.

$\lambda > 15$, so Y ~ N(25, 25) approximately
Continuity correction: P(Y > 25.5)
z = 0.100, so probability = 0.460
Answer: 0.460

3 Surds: sums of fractions (AQA, Edexcel, OCR, MEI, Eduqas)

Show that $\frac{2}{\sqrt{2}-1} + \frac{5}{\sqrt{2}+1}$ can be written as $p + q\sqrt{2}$, and find p and q.

Common denominator $(\sqrt{2}-1)(\sqrt{2}+1) = 2 - 1 = 1$
Numerator = $2(\sqrt{2}+1) + 5(\sqrt{2}-1)$
$= -3 + 7\sqrt{2}$
Divide by 1.
Answer: p = −3, q = 7

4 Double angle equations (All boards)

Solve $\cos 2x - 4\sin x = -5$
for $-180° \leq x \leq 180°$.

$\cos 2x \equiv 1 - 2\sin^2 x$: $2\sin^2 x + 4\sin x - 6 = 0$
Solve the quadratic: $\sin x = 1$ or $\sin x = -3$
$\sin x = 1$: $x = 90°$
$\sin x = -3$ is impossible
Answer: x = 90°

5 Interpolation for median and percentiles (All boards)

The table summarises the mass, m kg, of parcels of 61 items.
Use linear interpolation to estimate the lower quartile.

Position = 0.25 × 61 = 15.25
Cumulative frequency before the class 20 ≤ m < 30 is 14
Estimate = $20 + \frac{15.25 - 14}{14} \times 10$
= 20.9
Answer: 20.9

6 Equation after a sequence of transformations (All boards)

The curve with equation $y = x^2 + 2x + 5$ is transformed by a stretch, scale factor 3, parallel to the y-axis, followed by a reflection in the y-axis.
Find the equation of the image curve.

Stretch parallel to y: multiply by 3: $y = 3x^2 + 6x + 15$
Reflect in y-axis: replace x by −x: $y = 3x^2 - 6x + 15$
Answer: $y = 3x^2 - 6x + 15$

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7 Newton's second law with a time-varying force (AQA, Edexcel, OCR, MEI)

A 6 kg particle moves on a smooth horizontal line. At time t s a single force of $(24 - 4t)$ N acts along the line. When t = 0 its speed is 4 m s^{-1}. Find (a) its speed when t = 3, (b) its greatest speed.

F = ma: $a = (-4t + 24) \div 6$

$v = \int a\,dt = 4t - \frac{1}{3}t^2 + c$; v = 4 when t = 0

(a) v(3) = 13
(b) Greatest speed when a = 0: t = 6
v(6) = 16
Answer: (a) 13 m s^{-1} (b) 16 m s^{-1}

8 Integration by parts (All boards)

Use integration by parts to find the exact value of

$\int_1^e x^3 \ln x\,dx$.

$u = \ln x$, $\frac{dv}{dx} = x^3$: $\int x^3 \ln x\,dx = [\frac{x^4 \ln x}{4}] - \int \frac{x^3}{4}\,dx$

$= [\frac{x^4 \ln x}{4} - \frac{x^4}{16}]$

$= \frac{3e^4 + 1}{16}$

Answer: $\frac{3e^4 + 1}{16}$

9 Tangent conditions (All boards)

The line $y = k - 4x$ is a tangent to the curve $y = \frac{4}{x}$.

Find the possible values of k.

$\frac{4}{x} = k - 4x$

$4 = kx - 4x^2$, so $4x^2 - kx + 4 = 0$

Tangent: discriminant $k^2 - 4 \times 4 \times 4 = 0$

$k^2 = 64$

Answer: k = 8 or k = −8

10 Range of horizontal force on a rough slope (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 3 kg particle is in equilibrium on a rough plane inclined at α, $\tan\alpha = \frac{15}{8}$, μ = 0.5. A horizontal force of P N, in the vertical plane through a line of greatest slope, pushes it towards the plane. Find the least and greatest values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{15}{17}$, $\cos\alpha = \frac{8}{17}$, weight W = 29.4 N. Normal reaction $R = W\cos\alpha + P\sin\alpha$

Least P: about to slide down, friction F = μR up the slope
$P\cos\alpha + \mu(W\cos\alpha + P\sin\alpha) = W\sin\alpha \rightarrow P = W(\sin\alpha - \mu\cos\alpha) \div (\cos\alpha + \mu\sin\alpha) = 20.9$
Greatest P: about to slide up, friction down the slope
$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha) \rightarrow P = W(\sin\alpha + \mu\cos\alpha) \div (\cos\alpha - \mu\sin\alpha) = 1120$
Answer: least 20.9 N, greatest 1120 N