Day 162 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(39, p)$. H_0: $p = 0.4$ is tested against H_1: $p > 0.4$ using a 5% significance level.
(a) Find the critical region. (b) State the probability of a Type I error.
(c) Find the probability of a Type II error when $p = 0.5$ (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \geq 22$
(b) Type I error = P(reject H_0 | $p = 0.4$) = 0.0280
(c) Type II error = P(not in critical region | $p = 0.5$)
$P(X \leq 21)$ when $p = 0.5$: 0.739
Answer: (a) $X \geq 22$ (b) 0.0280 (c) 0.739

2 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 + 3$ and $g(x) = 3x - 2$.
Solve $fg(x) = gf(x)$.

$fg(x) = (3x - 2)^2 + 3 = 9x^2 - 12x + 7$
$gf(x) = 3(x^2 + 3) - 2 = 3x^2 + 7$
$9x^2 - 12x + 7 = 3x^2 + 7$ so $6x^2 - 12x = 0$
Answer: $x = 0$ or $x = 2$

3 Finding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(\mu, \sigma^2)$, with $P(X < 82) = 0.3085$ and $P(X > 94) = 0.0668$.
Find μ and σ.

z-values: 82: $z = -0.500$; 94: $z = 1.50$
$82 = \mu - 0.500\,\sigma$ and $94 = \mu + 1.50\,\sigma$
Subtract: $12 = 2.00\,\sigma$, so $\sigma = 6.00$
$\mu = 82 + 0.500 \times 6.00 = 85.0$
Answer: $\mu = 85$, $\sigma = 6$

4 Fixed point iteration (All boards)

The equation $x^2 = 5x + 1$ has a root α. Use the iteration $x_{n+1} = \sqrt{5x + 1}$ with $x_0 = 2$
to find x_1, x_2 and x_3 to 4 d.p., and hence the value of α to 2 d.p. (continue until the 2 d.p. value is settled).

$x_1 = \sqrt{5 \times 2 + 1} = 3.3166$
$x_2 = 4.1932$, $x_3 = 4.6868$
Continuing, the values settle at 5.1926, so $\alpha = 5.19$
Answer: $x_1 = 3.3166$, $x_2 = 4.1932$, $x_3 = 4.6868$; $\alpha = 5.19$

5 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(4 - x)^{\frac{1}{2}}$ in ascending powers of x, up to and including x^2.
State the range of x for the expansion is valid.

$(4 - x)^{\frac{1}{2}} = 4^{\frac{1}{2}}(1 + \frac{-1x}{4})^{\frac{1}{2}}$

$4^{\frac{1}{2}} = 2$

Expand $(1 + y)^{\frac{1}{2}}$ with $y = \frac{-1x}{4}$

Valid for $|\frac{-1x}{4}| < 1$, i.e. $|x| < 4$

Answer: $2 - \frac{1}{4}x - \frac{1}{64}x^2$; $|x| < 4$

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6 Rough incline: find the coefficient of friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle slides from rest down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{3}{4}$. It moves 39.2 m in 5 s. Find (a) the coefficient of friction, (b) the speed after 5 s. Take g = 9.8.

$s = \frac{1}{2}at^2$: $39.2 = \frac{1}{2}\times a\times 5^2$, so $a = 3.136$
Along the slope: $mg\sin\alpha - \mu R = ma$, with $R = mg\cos\alpha$
$g(\sin\alpha - \mu\cos\alpha) = a$: $9.8(0.6 - 0.8\mu) = 3.136$
$0.6 - 0.8\mu = 0.32$, so $\mu = 0.35$
$v = at = 3.136\times 5$
Answer: $\mu = 0.35$; 15.68 m s^{-1}

7 Quotient rule: stationary points (All boards)

Find the coordinates of the stationary points of
$y = \frac{x}{x^2+4}$.

$$\frac{dy}{dx} = \frac{(x^2+4)\times 1 - x\times 2x}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2}$$

$\frac{dy}{dx} = 0$ when $x^2 = 4$, so $x = \pm 2$

$x = 2$: $y = \frac{2}{8} = \frac{1}{4}$; $x = -2$: $y = -\frac{1}{4}$

Answer: $(-2, -\frac{1}{4})$, $(2, \frac{1}{4})$

8 Range of horizontal force on a rough slope (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 3 kg particle is in equilibrium on a rough plane inclined at α, $\tan\alpha = \frac{8}{15}$, μ = 0.5. A horizontal force of P N, in the vertical plane through a line of greatest slope, pushes it towards the plane. Find the least and greatest values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{8}{17}$, $\cos\alpha = \frac{15}{17}$, weight W = 29.4 N. Normal reaction $R = W\cos\alpha + P\sin\alpha$
Least P: about to slide down, friction $F = \mu R$ up the slope
$P\cos\alpha + \mu(W\cos\alpha + P\sin\alpha) = W\sin\alpha \rightarrow P = W(\sin\alpha - \mu\cos\alpha) \div (\cos\alpha + \mu\sin\alpha) = 0.774$
Greatest P: about to slide up, friction down the slope
$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha) \rightarrow P = W(\sin\alpha + \mu\cos\alpha) \div (\cos\alpha - \mu\sin\alpha) = 41.4$
Answer: least 0.774 N, greatest 41.4 N

9 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix}-1\\-4\\-6\end{pmatrix} + t\begin{pmatrix}-3\\-3\\-3\end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(0, 2, −4) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathbf{PF} = \begin{pmatrix}-3t-1\\-3t-6\\-3t-2\end{pmatrix}$
$\mathbf{PF}\cdot\mathbf{d} = 0$ gives $27t + 27 = 0$, so $t = -1$
F = (2, −1, −3); $\mathbf{PF} = \begin{pmatrix}2\\-3\\1\end{pmatrix}$, so $|\mathbf{PF}| = \sqrt{14}$
Answer: F(2, −1, −3), distance $\sqrt{14}$

10 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 6 and the coefficient of x^2 is 15. Find n and k.

$nk = 6$ and $\frac{n(n-1)}{2}k^2 = 15$

$(n-1)k = \frac{2 \times 15}{6} = 5$ (dividing the second equation by the first)

$n\,k - (n-1)\,k = k = 6 - 5 = 1$

$n = \frac{6}{1} = 6$

Answer: n = 6, k = 1