Day 161 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 3 m s^{-1} and accelerates uniformly. She travels 40 m in the next 10 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $40 = 3\times10 + \frac{1}{2}\times a\times10^2$, so $a = 0.2$
(b) $v = u + at = 3 + 0.2\times10 = 5$
(c) For the midpoint, $s = 20$: $v^2 = u^2 + 2as = 3^2 + 2\times0.2\times20$
$v^2 = 17$
Answer: (a) 0.2 m s^{-2} (b) 5 m s^{-1} (c) 4.12 m s^{-1}

2 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $8^{3x-3} = 16^{2x+2}$.

Write both sides as powers of 2.
$(2^3)^{3x-3} = (2^4)^{2x+2}$
$3(3x - 3) = 4(2x + 2)$
$9x - 9 = 8x + 8$
$x = 17$
Answer: x = 17

3 Area between curves and the x-axis (with sketch) (All boards)

The diagram shows the curve C: $y = x(6 - x)$ and the line L: $y = 3x$.
Find the area of the shaded region between C and L.

Intersect: $x(6 - x) = 3x$ gives $x = 0$ or $x = 3$

Area $= \int(C - L)\,dx = \int_0^3 (3x - x^2)\,dx = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]$

$= \frac{3^3}{2} - \frac{3^3}{3} = \frac{9}{2}$

Answer: $\frac{9}{2}$

4 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{3} - 3) = 3 - 3\sqrt{3}$, writing x in the form $p + q\sqrt{3}$.

$x = \frac{3 - 3\sqrt{3}}{\sqrt{3} - 3}$

Multiply top and bottom by $\sqrt{3} + 3$.
Numerator: $(3 - 3\sqrt{3})(\sqrt{3} + 3) = -6\sqrt{3}$
Denominator: $3 - 9 = -6$
Answer: x = $\sqrt{3}$

5 Passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected vertically upwards from the ground. It passes a ledge 4 s after projection and again 7 s after projection. Find (a) the speed of projection, (b) the height of the ledge, (c) the greatest height. Take g = 9.8.

Same height at t = 4 and t = 7, so the stone is at its highest point at t = 5.5
(a) At the top $v = 0$: $0 = u - 9.8\times5.5$, so $u = 53.9$
(b) $s = ut - 4.9t^2$ with $t = 4$: $s = 53.9\times4 - 4.9\times4^2 = 137.2$
(c) $v^2 = u^2 - 2gs$: $0 = 53.9^2 - 19.6s$, $s = 148.225$
Answer: (a) 53.9 m s^{-1} (b) 137.2 m (c) 148.225 m

6 Normal approximation to the binomial (All boards)

$X \sim B(60, 0.3)$. Use a normal approximation to estimate $P(X \leq 19)$ to 3 significant figures.

$np = 18$, $npq = 12.6$, so $Y \sim N(18, 12.6)$
With continuity correction: $P(Y < 19.5)$
$z = \frac{19.5 - 18}{3.55} = 0.423$, $P(Z < 0.423) = 0.664$
Answer: 0.664

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7 Separable differential equations (All boards)

Solve the differential equation

$\frac{dy}{dx} = 2xy^2$,

given that y = 1 when x = 0. Give y in terms of x.

Separate: $\int \frac{1}{y^2}\,dy = \int 2x\,dx$, so $-\frac{1}{y} = 1x^2 + C$

x = 0, y = 1 gives $C = -\frac{1}{1}$

$-\frac{1}{y} = 1x^2 - \frac{1}{1}$

Answer: $\mathbf{y = \frac{1}{1 - x^2}}$

8 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

X ~ B(40, p). H_0: p = 0.5 is tested against H_1: p < 0.5 using a 5% significance level.

(a) Find the critical region. (b) State the probability of a Type I error.

(c) Find the probability of a Type II error when p = 0.35 (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: X ≤ 14

(b) Type I error = P(reject H_0 | p = 0.5) = 0.0403

(c) Type II error = P(not in critical region | p = 0.35)

P(X ≥ 15) when p = 0.35: 0.428

Answer: (a) X ≤ 14 (b) 0.0403 (c) 0.428

9 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{3x}{(1 + 2x)(1 + x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 + 2x)} + \frac{B}{(1 + x)}$: A = −3, B = 3

(b) $-3(1 + 2x)^{-1} = -3(1 - 2x + 4x^2 ...)$

Add the series for both fractions.

(c) Need |2x| < 1 and |x| < 1

Answer: (a) $\mathbf{\frac{-3}{(1 + 2x)} + \frac{3}{(1 + x)}}$ **(b)** $\mathbf{3x - 9x^2}$ **(c)** $\mathbf{|x| < \frac{1}{2}}$

10 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that

4sin 3θ + 4cos θ = 7.5

can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with 0 < θ < 0.8, to 3 s.f.

$\sin 3\theta \approx 3\theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$

$4 \times 3\theta + 4(1 - \frac{1}{2}\theta^2) = 7.5$

Multiply by 2 and rearrange: $4\theta^2 - 24\theta + 7 = 0$

$\theta = \frac{24 \pm \sqrt{464}}{8}$, so θ = 0.307

Answer: a = 4, b = −24, c = 7; $\mathbf{\theta \approx 0.307}$