Day 16 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Chain rule and gradients (All boards)

$y = \ln(3x^2 + 5)$.

Find $\frac{dy}{dx}$ and the gradient of the curve at $x = -1$.

$\frac{dy}{dx} = \frac{1}{3x^2 + 5} \times 6x = \frac{6x}{3x^2 + 5}$

At $x = -1$: gradient $= \frac{-6}{8} = -\frac{3}{4}$

Answer: $\frac{dy}{dx} = \frac{6x}{3x^2 + 5}$; **gradient** $= -\frac{3}{4}$

2 Catching up (delayed start) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes P at constant speed 20 m s^{-1}. 10 s later car B leaves P from rest, in the same direction, with acceleration 3 m s^{-2}. B overtakes A at Q. Find (a) the time B takes to reach Q, (b) PQ, (c) the speed of B at Q.

Let B take s seconds; A has then been moving for (s + 10) seconds.
B: $\frac{1}{2} \times 3 \times s^2 = 1.5\ s^2$ A: $20(s + 10)$
$1.5\ s^2 - 20\ s - 200 = 0$ so $s = 20$ ($s > 0$)
(b) $PQ = 1.5 \times 20^2 = 600$
(c) $v = 0 + at = 3 \times 20 = 60$
Answer: (a) 20 s (b) 600 m (c) 60 m s^{-1}

3 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 + 8x - 10y + 32 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P(−6, 5) lies inside or outside the circle.

Complete the square: $(x + 4)^2 + (y - 5)^2 = 9$
Centre (−4, 5), radius 3
Distance^2 from centre to P $= (-2)^2 + 0^2 = 4$
Compare with $r^2 = 9$
Answer: (a) centre (−4, 5), radius 3 (b) P is inside the circle

4 Laws of logarithms in terms of p and q (All boards)

Given that $\log_a 2 = p$ and $\log_a 3 = q$,

express $\log_a(\sqrt{12})$ in terms of p and q.

$\sqrt{12} = 2 \times 3^{\frac{1}{2}}$

$\log_a(2 \times 3^{\frac{1}{2}}) = \log_a 2 + \frac{1}{2} \log_a 3$

$= p + \frac{1}{2}q$

Answer: $\mathbf{p + \frac{1}{2}q}$

5 Inverse of a rational function (All boards)

$f(x) = \frac{4x + 6}{2x - 4}$, $x \in \mathbb{R}$, $x \neq 2$.

Find $f^{-1}(x)$ and state its domain.

$y = \frac{4x + 6}{2x - 4}$: $y(2x - 4) = 4x + 6$
$2xy - 4y = 4x + 6$
$x(2y - 4) = 4y + 6$
$x = \frac{4y + 6}{2y - 4}$, then swap x and y

Answer: $\mathbf{f^{-1}(x) = \frac{4x + 6}{2x - 4}}$, $\mathbf{x \neq 2}$

6 Outliers (AQA, Edexcel, OCR, MEI, Eduqas)

A data set has mean 66 and standard deviation 9.
A value is treated as an outlier if it is more than 2 standard deviations from the mean.
Which of 47, 51, 60, 86 are outliers?

2 standard deviations = 2 × 9 = 18
Lower limit = 66 − 18 = 48
Upper limit = 66 + 18 = 84
Outliers are values below 48 or above 84
Answer: 47, 86

7 Power, driving force and resistance (CIE)

A 1200 kg car has engine power 45 kW and constant resistance 500 N. Find (a) its greatest steady speed on a horizontal road, (b) its acceleration at 12 m s^{-1} up a hill where $\sin\alpha = \frac{1}{20}$. Take g = 10. Give answers to 3 s.f. if not exact.

(a) Steady speed: driving force = resistance. P = Fv: 45000 = 500 v
(b) Driving force F = P ÷ v = 45000 ÷ 12 = 3750 N
Up the hill: F − 500 − 1200 g sin α = 1200 a
3750 − 500 − 600 = 1200 a
$a = \frac{53}{24}$
Answer: (a) 90 m s^{-1} (b) 2.21 m s^{-2}

8 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
3sin θ + 4cos 2θ = 3.5
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin\theta \approx \theta$, $\cos 2\theta \approx 1 - \frac{4}{2}\theta^2$

$3 \times \theta + 4(1 - \frac{4}{2}\theta^2) = 3.5$

Multiply by 2 and rearrange: $16\theta^2 - 6\theta - 1 = 0$

$\theta = \frac{6 \pm \sqrt{100}}{32}$, so θ = 0.500
Answer: a = 16, b = −6, c = −1; θ ≈ 0.500

9 Two-tailed binomial test critical region (AQA, Edexcel, OCR, MEI, Eduqas)

X ~ B(37, p). H_0: p = 0.3 is tested against H_1: p ≠ 0.3 at the 10% level.
(a) Find the critical region.
(b) Find the actual significance level of the test (3 s.f.).

(a) Each tail ≤ 0.05. P(X ≤ 6) = 0.0440, P(X ≤ 7) = 0.0947
P(X ≥ 17) = 0.0295, P(X ≥ 16) = 0.0604
Critical region: X ≤ 6 or X ≥ 17
(b) Actual level = 0.0440 + 0.0295 = 0.0734
Answer: (a) X ≤ 6 or X ≥ 17 (b) 0.0734

10 Exponential equations: quadratic in b^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3 \times 2^{2x} - 9 \times 2^x + 6 = 0$.

Let $y = 2^x$. Then $3y^2 - 9y + 6 = 0$
3(y − 1)(y − 2) = 0
y = 1 or y = 2
$2^x = 1$ gives x = 0; $2^x = 2$ gives x = 1
Answer: x = 0 or x = 1