Day 159 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Chain rule and gradients (All boards)

$y = (x - 5)^3$.

Find $\frac{dy}{dx}$ and the gradient of the curve at $x = 1$.

Let $u = x - 5$, so $y = u^3$, $\frac{du}{dx} = 1$, $\frac{dy}{du} = 3u^2$

$\frac{dy}{dx} = 3(x - 5)^2$

At $x = 1$: $u = -4$, so gradient = 48

Answer: $\frac{dy}{dx} = 3(x - 5)^2$; gradient = 48

2 Sigma notation (All boards)

Find the value of $\sum_{r=1}^{8} 3 \times 4^r$.

Geometric series: first term 12, ratio 4, 8 terms

$S = \frac{12(4^8 - 1)}{(4 - 1)}$

$= \frac{12 \times 65535}{3}$

Answer: 262140

3 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^{\ln 2} (5e^{2x} + e^x)\,dx$.

Integrate: $[\frac{5}{2}e^{2x} + e^x]$

Substitute the limits and subtract: $\frac{17}{2}$

Answer: $\frac{17}{2}$

4 Selections with combinations (OCR, MEI, Eduqas, CIE)

A group of 6 boys and 7 girls. A team of 3 is chosen at random.
Find the probability that the team has exactly 2 of the girls.

Total teams = C(13, 3) = 286

Favourable = C(7, 2) × C(6, 1) = 21 × 6 = 126

$P = \frac{126}{286}$

$= \frac{63}{143}$

Answer: $\frac{63}{143}$

5 Work-energy on a rough slope (CIE)

A 5 kg block is released from rest on a rough plane inclined at α, $\tan\alpha = \frac{5}{12}$, $\mu = 0.2$, and slides 4 m down a line of greatest slope. Use work and energy to find (a) the work done against friction, (b) its speed after 4 m. Take $g = 10$. Give answers to 3 s.f. if not exact.

$R = 5g\cos\alpha = \frac{600}{13}$ N, $F = \mu R = \frac{120}{13}$ N

(a) Work done against friction $= F \times d = \frac{120}{13} \times 4$

(b) Loss of PE $= 5g \times 4\sin\alpha = \frac{1000}{13}$ J

Work-energy: $\frac{1}{2}\times 5 \times v^2 = \frac{1000}{13} - \frac{480}{13} = 40$

$v^2 = 16$

Answer: (a) 36.9 J (b) 4 m s^{-1}

6 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $12x + 5y = k$ is a tangent to the circle $(x + 4)^2 + (y + 4)^2 = 9$.
Find the two possible values of k.

Centre $(-4, -4)$, radius 3
Distance from centre to the line = radius:

$\frac{|-68 - k|}{13} = 3$

$|-68 - k| = 39$

Answer: k = −107 or k = −29

7 Regression with coded variables (AQA, Edexcel, OCR, MEI, Eduqas)

A regression line of v on w is $v = 1001 + 5w$, where $w = \frac{t - 2010}{10}$.

(a) Find the regression line of v on t in the form $v = p + qt$.
(b) Estimate v when $t = 2050$.

$v = 1001 + 5 \times \frac{t - 2010}{10}$

$= 1001 + 0.5t - 1005 = -4 + 0.5t$

(b) $v = -4 + 0.5 \times 2050 = 1021$

Answer: (a) v = −4 + 0.5t (b) 1021

8 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (1 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (9 kg), hanging 1.2 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

B: $9g - T = 9a$
A: $T = 1a$
Adding: $9g = 10a$, so $a = 8.82$
(a) $T = 1 \times 8.82 = 8.82$
(b) $v^2 = 0 + 2\times 8.82 \times 1.2 = 21.168$

Answer: (a) 8.82 N (b) 4.60 m s^{-1}

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9 Equations with cosec x and cot x (All boards)

Solve $3\text{cosec}^2 x - 6\cot x - 12 = 0$
for $-180° < x < 180°$, $x \neq 0$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $3\cot^2 x - 6\cot x - 9 = 0$
$\cot x = -1$ or $\cot x = 3$
$\cot x = -1$ means $\tan x = -1$: $x = -45.0°, 135.0°$
$\cot x = 3$ means $\tan x = \frac{1}{3}$: $x = -161.6°, 18.4°$

Answer: $x = -161.6°, -45.0°, 18.4°, 135.0°$

10 Line and curve intersection (diagram) (All boards)

The diagram shows the curve $y = x^2 + 3x - 8$ and the line $y = 2x - 2$, meeting at A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the midpoint of AB.

$x^2 + 3x - 8 = 2x - 2$: $x^2 + x - 6 = 0$
$(x + 3)(x - 2) = 0$, $x = -3$ or 2
$A(-3, -8)$, $B(2, 2)$
Midpoint: average the coordinates

Answer: (a) $A(-3, -8)$, $B(2, 2)$ (b) $(-\frac{1}{2}, -3)$