Day 158 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Outliers (AQA, Edexcel, OCR, MEI, Eduqas)

A data set has mean 70 and standard deviation 3.
A value is treated as an outlier if it is more than 3 standard deviations from the mean.
Which of 58, 75, 83, 85 are outliers?

3 standard deviations = 3 × 3 = 9
Lower limit = 70 − 9 = 61
Upper limit = 70 + 9 = 79
Outliers are values below 61 or above 79
Answer: 58, 83, 85

2 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 24.5 m s^{-1} from a point h m above horizontal ground, and hits the ground 6 s later.
Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: s = −h, u = 24.5, a = −9.8, t = 6
$-h = 24.5\times6 - 4.9\times6^2$, so h = 29.4
(b) At the highest point v = 0: $0 = 24.5^2 - 2\times9.8\times s$
s = 30.625 m above the point of projection
Greatest height above the ground = 29.4 + 30.625
Answer: (a) 29.4 m (b) 60.025 m

3 Chain rule and gradients (All boards)

$y = \ln(x^2 + 4)$.
Find $\frac{dy}{dx}$ and the gradient of the curve at x = −1.

$$\frac{dy}{dx} = \frac{1}{x^2+4} \times 2x = \frac{2x}{x^2+4}$$

At x = −1: gradient = $\frac{-2}{5} = -\frac{2}{5}$

Answer: $\frac{dy}{dx} = \frac{2x}{x^2+4}$; gradient = $-\frac{2}{5}$

4 Arithmetic series: S_n formula (All boards)

The sum of the first n terms of an arithmetic series is $S_n = n^2 - 3n$.
(a) Find the first term and the common difference.
(b) Find u_{24}.

$u_1 = S_1 = -2$
$S_2 = -2$, so $u_2 = 0$
$d = u_2 - u_1 = 2$
$u_{24} = -2 + 23 \times 2$
Answer: (a) a = −2, d = 2 (b) 44

5 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 2050 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(120 + 85) × V
2050 = 102.5 V, so V = 20
(b) Deceleration = V ÷ 20 = 20 ÷ 20
Answer: (a) V = 20 (b) 1 m s^{-2}

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6 Equations with sec x and tan x (All boards)

Solve $3\sec^2 x - 6\tan x - 12 = 0$
for $0° \le x \le 180°$. Give answers to 1 d.p.

Use $\sec^2 x \equiv 1 + \tan^2 x$: $3\tan^2 x - 6\tan x - 9 = 0$
$\tan x = -1$ or $\tan x = 3$
$\tan x = -1$: $x = 135.0°$
$\tan x = 3$: $x = 71.6°$
Answer: $x = 71.6°, 135.0°$

7 Venn diagram with algebra (All boards)

The Venn diagram shows the numbers of 49 students in sets A and B.
(a) Find the value of x.
(b) A student is chosen at random. Find P(A′ | B′).

(a) Total: $(3x + 4) + (x + 3) + (2x + 1) + 5 = 49$
$6x + 13 = 49$, so $x = 6$
(b) Regions: 22, 9, 13, 5
Not in B: 27, of which not in A: 5
Answer: (a) $x = 6$ (b) $\frac{5}{27}$

8 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that $\sqrt{5}$ is irrational.

Assume $\sqrt{5} = \frac{a}{b}$ with a, b integers with no common factor.

Then $5b^2 = a^2$, so a^2 is a multiple of 5, hence a is a multiple of 5.
Write $a = 5c$. Then $5b^2 = 25c^2$, so $b^2 = 5c^2$.
So b is also a multiple of 5, contradicting no common factor.
Answer: contradiction, so $\sqrt{5}$ is irrational

9 Optimisation problems (All boards)

An open-topped box has a square base of side x cm and volume 4 cm^3.

Show that its surface area is $A = x^2 + \frac{16}{x}$, and find the least possible value of A, justifying that it is a minimum.

$h = \frac{4}{x^2}$, so $A = x^2 + 4xh = x^2 + \frac{16}{x}$

$\frac{dA}{dx} = 2x - \frac{16}{x^2} = 0$, so $x^3 = 8$ and $x = 2$

$\frac{d^2A}{dx^2} = 2 + \frac{32}{x^3} > 0$, so a minimum

$A = 4 + \frac{16}{2} = 12$

Answer: $A_{min} = 12\ cm^2$ ($x = 2$)

10 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(4 - 3x)^{-2}$ in ascending powers of x, up to and including x^2.
State the range of x for which the expansion is valid.

$(4 - 3x)^{-2} = 4^{-2}(1 + \frac{-3x}{4})^{-2}$

$4^{-2} = \frac{1}{16}$

Expand $(1 + y)^{-2}$ with $y = \frac{-3x}{4}$

Valid for $|\frac{-3x}{4}| < 1$, i.e. $|x| < \frac{4}{3}$

Answer: $\frac{1}{16} + \frac{3}{32}x + \frac{27}{256}x^2$; $|x| < \frac{4}{3}$