**Day 157** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Polynomial division (All boards)

Find the quotient and the remainder when $x^4 - 2x - 17$ is divided by $(x^2 + 4)$.

Quotient (matching leading terms in turn): $x^2 - 4$

$(x^2 + 4)(x^2 - 4) = x^4 - 16$

Remainder = $x^4 - 2x - 17 - (x^4 - 16) = -2x - 1$

**Answer: quotient $x^2 - 4$, remainder $-2x - 1$**

## 2 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against ln x for $y = A x^n$.
Find the value of n and the exact value of A.

ln y = ln A + n ln x: gradient n, intercept ln A

$n = \frac{9 - 7}{3 - 2} = 2$

At (2, 7): ln A = 7 − 2 × 2 = 3, so $A = e^3$

**Answer: n = 2, A = $e^3$**

## 3 Binomial expansion: a coefficient (All boards)

Find the coefficient of $x^2$ in the expansion of $(-1 + 4x)^8$.

Term in $x^2$: ${}^8C_2 \times (-1)^6 \times (4x)^2$

= 28 × 1 × 16

**Answer: 448**

## 4 Power, driving force and resistance (CIE)

A 1000 kg car has engine power 40 kW and constant resistance 1000 N. Find (a) its greatest steady speed on a horizontal road, (b) its acceleration at 24 m $s^{-1}$ up a hill where $\sin\alpha = \frac{1}{20}$. Take g = 10. Give answers to 3 s.f. if not exact.

(a) Steady speed: driving force = resistance. P = Fv: 40000 = 1000 v

(b) Driving force $F = P \div v = 40000 \div 24 = \frac{5000}{3}$ N

Up the hill: $F - 1000 - 1000\,g \sin\alpha = 1000\,a$

$\frac{5000}{3} - 1000 - 500 = 1000\,a$

$a = \frac{1}{6}$

**Answer: (a) 40 m $s^{-1}$ (b) 0.167 m $s^{-2}$**

## 5 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For n = 9 pairs of (x, y): $\Sigma x = 130$, $\Sigma y = 461$,
$\Sigma x^2 = 2300$, $\Sigma y^2 = 25845$, $\Sigma xy = 7586$.
Calculate the product moment correlation coefficient, and describe the correlation.

$S_{xx} = 2300 - \frac{130^2}{9} = 422$

$S_{yy} = 25845 - \frac{461^2}{9} = 2230$

$S_{xy} = 7586 - \frac{130 \times 461}{9} = 927$

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = 0.955$

This is strong positive correlation

**Answer: r = 0.955, positive correlation**

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## 6 Exponential equations reducing to quadratics (All boards)

Solve $e^x + 5e^{-x} = 6$, giving exact answers.

Multiply by $e^x$: $e^{2x} + 5 = 6e^x$
Let $u = e^x$: $u^2 - 6u + 5 = 0$, so $(u - 1)(u - 5) = 0$
$e^x = 1$ or $e^x = 5$
**Answer: $x = 0$ or $x = \ln 5$**

## 7 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle $\alpha$ above the horizontal, $\tan\alpha = \frac{3}{4}$, and passes through A, 48 m horizontally from O and 16.4 m above the ground. Find the speed of projection. Take $g = 9.8$.

Let the speed be U. Horizontal: $48 = U\cos\alpha \times t$, so $t = 48 \div (0.8\,U)$
Vertical: $16.4 = U\sin\alpha \times t - 4.9t^2$
Eliminate t: $y = x\tan\alpha - 4.9x^2 \div (U^2\cos^2\alpha)$
$16.4 = 36 - 4.9\times48^2 \div (0.64\,U^2)$
$U^2 = 17640 \div 19.6 = 900$, $U = 30$
**Answer: 30 m s$^{-1}$**

## 8 Normal approximation to the Poisson (CIE)

$X \sim Po(18)$. Use a suitable approximation to find $P(X \geq 13)$, to 3 significant figures.

$\lambda > 15$, so $Y \sim N(18, 18)$ approximately
Continuity correction: $P(Y > 12.5)$
$z = -1.30$, so probability = 0.903
**Answer: 0.903**

## 9 Sum to infinity: two possible ratios (All boards)

The second term of a geometric series is 10 and its sum to infinity is 45.
Find the two possible values of the first term.

$ar = 10$ and $\frac{a}{(1-r)} = 45$
$a = 45(1 - r)$, so $45(1 - r)\,r = 10$
$45r^2 - 45r + 10 = 0$: $r = \frac{1}{3}$ or $r = \frac{2}{3}$
$a = 45(1 - r) = 30$ or 15
**Answer: $a = 15$ or $a = 30$**

## 10 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

$l_1$: $\mathbf{r} = \begin{pmatrix}-2\\10\\-7\end{pmatrix} + \lambda\begin{pmatrix}2\\-3\\2\end{pmatrix}$
$l_2$: $\mathbf{r} = \begin{pmatrix}-5\\-2\\-10\end{pmatrix} + \mu\begin{pmatrix}3\\1\\3\end{pmatrix}$

Determine whether $l_1$ and $l_2$ intersect. If they do, find the point of intersection.

Equate components: $-2 + 2\lambda = -5 + 3\mu$; $10 - 3\lambda = -2 + 1\mu$; $-7 + 2\lambda = -10 + 3\mu$
Solving: $\lambda = 3$, $\mu = 3$, and all three equations are satisfied
Point of intersection = (4, 1, −1)
**Answer: Intersect at (4, 1, −1)**