Day 156 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Transformations of a turning point (All boards)

The curve y = f(x) has a maximum point at (−2, −2).
Find the coordinates of the corresponding point on:
(a) y = f(x) + 2
(b) y = −f(x)
(c) y = f(−x)

(a) y = f(x) + 2: (−2, 0)
(b) y = −f(x): (−2, 2)
(c) y = f(−x): (2, −2)
A stretch or translation moves the point in the stated direction only.
Answer: (a) (−2, 0); (b) (−2, 2); (c) (2, −2)

2 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The life of a battery is N(70, 8^2). A battery is classed as 'special' if its life is more than 55.
10 batterys are chosen at random. Find the probability that exactly 3 are special (3 s.f.).

P(special) = P(X > 55) = P(Z > −1.88) = 0.970
Y ~ B(10, 0.970)
P(Y = 3) = C(10, 3)$(0.970)^3(0.0304)^7$ = 0.00000000262
Answer: 0.00000000262

3 Discriminant: range of k (All boards)

The equation x^2 + (k)x + (2k) = 0 has no real roots.
Find the set of possible values of k.

Discriminant: $b^2 - 4ac = (k)^2 - 4(2k)$
$= k^2 - 8k$
Roots of the quadratic in k: k = 0 and k = 8
Required: $b^2 - 4ac < 0$
Answer: 0 < k < 8

4 Collinear points and ratios (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A, B and C have position vectors $\begin{pmatrix}-3\\3\\0\end{pmatrix}$, $\begin{pmatrix}-5\\4\\-2\end{pmatrix}$ and $\begin{pmatrix}-1\\k\\2\end{pmatrix}$
relative to O, where k is a constant. Given that A, B and C are collinear, find k and the ratio AB : BC.

AB = $\begin{pmatrix}-2\\1\\-2\end{pmatrix}$, and AC = $\begin{pmatrix}2\\k-3\\2\end{pmatrix}$
AC = λAB: component 2 gives −1 = 1 λ, so λ = −1
Hence k = 3 + (−1) × 1 = 2
AC = −1 AB, so BC = −2 AB: AB : BC = 1 : 2
Answer: k = 2, AB : BC = 1 : 2

5 Interpolation for median and percentiles (All boards)

The table summarises the mass, m kg, of parcels of 51 items.
Use linear interpolation to estimate the 90th percentile.

Position = 0.9 × 51 = 45.9
Cumulative frequency before the class 40 ≤ m < 50 is 45
Estimate = $40 + \frac{45.9 - 45}{6} \times 10$
= 41.5
Answer: 41.5

6 Non-uniform rod and equal tensions (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB (length 2 m, mass 4 kg) has its centre of mass at G, AG = 0.25 m. It hangs horizontally from vertical strings at A and B. A particle of mass m kg is fixed at P, AP = 1.5 m, making the tensions equal. Find (a) m, (b) the tension. Take g = 9.8.

Equal tensions T: $2T = (4 + m)g$, so $T = \frac{1}{2}(4 + m)g$
Moments about A: $T \times 2 = 4g \times 0.25 + mg \times 1.5$
$\frac{1}{2}(4 + m) \times 2 = 1 + 1.5m \rightarrow 4 + 1m = 1 + 1.5m$
$m(1.5 - 1) = 4 - 1$, so $m = 6$
(b) $T = \frac{1}{2}(4 + 6) \times 9.8$
Answer: (a) m = 6 (b) 49 N

7 Binomial expansion: terms with negative powers of x (All boards)

Find the term independent of x in the expansion of
$(2x^3 + \frac{3}{x^2})^5$.

General term: ${}^5C_r\,(2x^3)^{5-r}\,(3x^{-2})^r$
Power of x: $3(5 - r) - 2r = 0$
$15 - 5r = 0$ so $r = 3$
Coefficient = ${}^5C_3 \times 2^2 \times 3^3 = 1080$
Answer: 1080

8 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 10 \\ 7 \end{pmatrix} + t\begin{pmatrix} 0 \\ -3 \\ -1 \end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(1, 1, 4) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathbf{PF} = \begin{pmatrix} -1 \\ -3t + 9 \\ -t + 3 \end{pmatrix}$
$\mathbf{PF} \cdot \mathbf{d} = 0$ gives $10t - 30 = 0$, so $t = 3$
F = (0, 1, 4); $\mathbf{PF} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$, so $|\mathbf{PF}| = \sqrt{1} = 1$
Answer: F(0, 1, 4), distance 1

9 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (16 kg, 3 m) is smoothly hinged at A to a vertical wall, with a 3 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, $\tan\theta = \frac{4}{3}$. Find (a) the tension, (b) the magnitude of the hinge force. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about A: $T\sin\theta \times 3 = 16g \times 1.5 + 3g \times 3$
$T\sin\theta = (8 + 3)g = 107.8$, so (a) $T = 135$
Horizontal component of hinge force: $X = T\cos\theta = 80.9$
Vertical: $Y + T\sin\theta = (16 + 3)g$, so $Y = 16g \div 2 = 78.4$
(b) Force $= \sqrt{(X^2 + Y^2)} = \sqrt{(80.9^2 + 78.4^2)}$
Answer: (a) 135 N (b) 113 N

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10 Area and perimeter of a segment (exact) (All boards)

The circle has centre O and radius 18 cm. Angle AOB = $\frac{\pi}{2}$.

Find the exact area and the exact perimeter of the shaded segment.

Sector area = $\frac{1}{2}r^2\theta = 81\pi$; triangle OAB = $\frac{1}{2}r^2\sin\theta = 162$

Segment area = sector − triangle = $81\pi - 162$

Chord AB = $2r\sin\frac{\theta}{2} = 18\sqrt{2}$; arc = $r\theta = 9\pi$

Perimeter = arc + chord = $18\sqrt{2} + 9\pi$

Answer: area = $81\pi - 162$ cm^2, perimeter = $18\sqrt{2} + 9\pi$ cm