Day 155 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Parametric differentiation: tangent and normal (All boards)

A curve has parametric equations $x = t^3$, $y = 3t^2$.

Find $\frac{dy}{dx}$ in terms of t, and the equation of the normal at the point where $t = -1$, in the form $ax + by + c = 0$ (integers).

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{2}{t}$

$t = -1$: point $(-1, 3)$, gradient -2

Normal gradient $= -1 \div (-2) = \frac{1}{2}$, so normal: $x - 2y + 7 = 0$

Answer: $\frac{dy}{dx} = \frac{2}{t}$; $x - 2y + 7 = 0$

2 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 14.7 m s^{-1} from a point h m above horizontal ground, and hits the ground 4 s later. Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: $s = -h$, $u = 14.7$, $a = -9.8$, $t = 4$

$-h = 14.7 \times 4 - 4.9 \times 4^2$, so $h = 19.6$

(b) At the highest point $v = 0$: $0 = 14.7^2 - 2 \times 9.8 \times s$

$s = 11.025$ m above the point of projection

Greatest height above the ground = 19.6 + 11.025

Answer: (a) 19.6 m (b) 30.625 m

3 Modelling with a normal distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

At one location, the daily mean wind speed is modelled by $N(7.8, 2.9^2)$.

(a) Find the probability that on a randomly chosen day the value exceeds 12 knots.

(b) Estimate the number of such days in a period of 92 days.

(a) $z = \frac{12 - 7.8}{2.9} = 1.45$

$P(Z > 1.45) = 0.0738$

(b) $92 \times 0.0738 = 6.79 \approx 7$

Answer: (a) 0.0738 (b) 7 days

4 Double angle equations (All boards)

Solve $\cos 2\theta + 5\cos\theta = -4$ for $0° \leq \theta \leq 360°$.

$\cos 2\theta \equiv 2\cos^2\theta - 1$: $2\cos^2\theta + 5\cos\theta + 3 = 0$

Solve the quadratic: $\cos\theta = -1$ or $\cos\theta = -\frac{3}{2}$

$\cos\theta = -1$: $\theta = 180°$

$\cos\theta = -\frac{3}{2}$ is impossible

Answer: $\theta = 180°$

5 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $-3x + 4y = k$ is a tangent to the circle $(x + 2)^2 + (y - 3)^2 = 4$.

Find the two possible values of k.

Centre $(-2, 3)$, radius 2

Distance from centre to the line = radius:

$\frac{|18 - k|}{5} = 2$

$|18 - k| = 10$

Answer: k = 8 or k = 28

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6 Rough incline: find the coefficient of friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle slides from rest down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{3}{4}$. It moves 44.1 m in 5 s. Find (a) the coefficient of friction, (b) the speed after 5 s. Take $g = 9.8$.

$s = \frac{1}{2}at^2$: $44.1 = \frac{1}{2}\times a\times 5^2$, so $a = 3.528$
Along the slope: $mg\sin\alpha - \mu R = ma$, with $R = mg\cos\alpha$
$g(\sin\alpha - \mu\cos\alpha) = a$: $9.8(0.6 - 0.8\mu) = 3.528$
$0.6 - 0.8\mu = 0.36$, so $\mu = 0.3$
$v = at = 3.528\times 5$
Answer: $\mu = 0.3$; 17.64 m s^{-1}

7 Conditional probability and independence (All boards)

$P(B) = 0.5$, $P(A) = 0.1$ and $P(A \mid B) = 0.1$.
(a) Find $P(A \cap B)$ and $P(A \cup B)$.
(b) Find $P(B \mid A)$.
(c) Determine whether A and B are independent.

(a) $P(A \cap B) = P(A \mid B)P(B) = 0.1 \times 0.5 = 0.05$
$P(A \cup B) = 0.1 + 0.5 - 0.05 = 0.55$
(b) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.05}{0.1} = 0.5$
(c) $P(A)P(B) = 0.1 \times 0.5 = 0.05$, which equals $P(A \cap B) = 0.05$
So A and B are independent
Answer: (a) 0.05, 0.55 (b) 0.5 (c) independent

8 Sum to infinity: two possible ratios (All boards)

The second term of a geometric series is 4 and its sum to infinity is 18.
Find the two possible values of the first term.

$ar = 4$ and $\frac{a}{(1-r)} = 18$
$a = 18(1 - r)$, so $18(1 - r)\,r = 4$
$18r^2 - 18r + 4 = 0$: $r = \frac{1}{3}$ or $r = \frac{2}{3}$
$a = 18(1 - r) = 12$ or 6
Answer: $a = 6$ or $a = 12$

9 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix}6\\4\\1\end{pmatrix} + \lambda\begin{pmatrix}0\\2\\-1\end{pmatrix}$
l_2: $\mathbf{r} = \begin{pmatrix}6\\0\\-6\end{pmatrix} + \mu\begin{pmatrix}0\\2\\2\end{pmatrix}$
Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $6 + 0\lambda = 6 + 0\mu$; $4 + 2\lambda = 0 + 2\mu$; $1 - 1\lambda = -6 + 2\mu$
Solving: $\lambda = 1$, $\mu = 3$, and all three equations are satisfied
Point of intersection = (6, 6, 0)
Answer: Intersect at (6, 6, 0)

10 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point $(-6, 12)$. Its centre lies in the second quadrant. Find the equations of the two possible circles.

Centre $(-r, r)$, radius $r > 0$.
Through $(-6, 12)$: $(-6 + r)^2 + (12 - r)^2 = r^2$
$r^2 - 36r + 180 = 0$
$(r - 6)(r - 30) = 0$, so $r = 6$ or 30
Answer: $(x + 6)^2 + (y - 6)^2 = 36$ or $(x + 30)^2 + (y - 30)^2 = 900$