Day 154 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Show by counter-example that the statement "|x + y| = |x| + |y| for all real x and y" is false.

Take x = 2 and y = −3:
|x + y| = |−1| = 1, but |x| + |y| = 2 + 3 = 5.
Answer: x = 2, y = −3

2 Smooth incline held by a string (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 2 kg particle rests on a smooth plane inclined at α, $\tan\alpha = \frac{5}{12}$, held by a string in the vertical plane through a line of greatest slope, at angle θ above the slope, $\tan\theta = \frac{4}{3}$. Find the tension and the normal reaction. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$, $\sin\theta = 0.8$, $\cos\theta = 0.6$
Along the plane: $T\cos\theta = 2g\sin\alpha$, so T = 12.6
Perpendicular: $R + T\sin\theta = 2g\cos\alpha$
$R = 2g\cos\alpha - T\sin\theta = 18.1 - 12.6\times0.800$
Answer: T = 12.6 N, R = 8.04 N

3 Arithmetic series: finding n (All boards)

An arithmetic series has first term −8 and common difference 2.
The sum of the first n terms is 136. Find n.

$\frac{n}{2}(2\times(-8) + (n-1)\times 2) = 136$

$2n^2 - 18n = 272$

$2n^2 - 18n - 272 = 0$

$(n - 17)(2n + 16) = 0$, so n = 17 or n = −8
n must be a positive integer, so n = 17
Answer: 17

4 Product rule: stationary points (All boards)

$y = (2x + 5)e^{-2x}$.
Find the exact coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 2e^{-2x} - 2(2x + 5)e^{-2x} = (-4x - 8)e^{-2x}$

$e^{-2x} > 0$, so $\frac{dy}{dx} = 0$ gives $-4x - 8 = 0$ and x = −2

$y = e^4$; $\frac{d^2y}{dx^2}$ is negative there, so it is a maximum

Answer: $(-2, e^4)$, maximum

5 Equation after a sequence of transformations (All boards)

The curve with equation $y = -x^2 - 3x - 1$ is transformed by a stretch, scale factor 2, parallel to the y-axis, followed by a stretch, scale factor 3, parallel to the x-axis.
Find the equation of the image curve.

Stretch parallel to y: multiply by 2: $y = -2x^2 - 6x - 2$

Stretch parallel to x: replace x by $\frac{x}{3}$: $y = -\frac{2}{9}x^2 - 2x - 2$

Answer: $y = -\frac{2}{9}x^2 - 2x - 2$

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6 Selections with combinations (OCR, MEI, Eduqas, CIE)

A group of 9 men and 8 women. A team of 3 is chosen at random.
Find the probability that the team has exactly 1 of the women.

Total teams = C(17, 3) = 680
Favourable = C(8, 1) × C(9, 2) = 8 × 36 = 288
$P = \frac{288}{680}$
$= \frac{36}{85}$

Answer: $\mathbf{\frac{36}{85}}$

7 Greatest and least values using R-form (All boards)

$g(\theta) = \frac{1}{13 + 3\sin\theta + 4\cos\theta}$
(a) Write $3\sin\theta + 4\cos\theta$ in the form $R\sin(\theta + \alpha)$, with α acute.
(b) Find the greatest and least values of $g(\theta)$ and the smallest positive θ at which the greatest value occurs (1 d.p.).

$R = \sqrt{3^2 + 4^2} = 5$, $\alpha = \tan^{-1}\frac{4}{3} = 53.1°$
Denominator = $13 + 5\sin(\theta + 53.1°)$, ranging from 8 to 18
Greatest g = $\frac{1}{8}$ (denominator least), least g = $\frac{1}{18}$
Least denominator when $\sin(\theta + 53.1°) = -1$: $\theta + 53.1° = 270°$
$\theta = 216.9°$

Answer: R = 5, α = 53.1°; greatest $\frac{1}{8}$, least $\frac{1}{18}$; θ = 216.9°

8 Normal approximation to the Poisson (CIE)

X ~ Po(30). Use a suitable approximation to find $P(X \geq 38)$, to 3 significant figures.

$\lambda > 15$, so Y ~ N(30, 30) approximately
Continuity correction: P(Y > 37.5)
z = 1.37, so probability = 0.0855
Answer: 0.0855

9 Smooth incline: passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected up a line of greatest slope of a smooth plane inclined at α. It passes A, 23.52 m from the start, after 4 s and again after 6 s. Find (a) the speed of projection, (b) α to 1 d.p. Take g = 9.8.

Deceleration $a = g\sin\alpha$ (unknown). $s = ut - \frac{1}{2}at^2$ at both times:
$u\times4 - \frac{1}{2}a\times4^2 = u\times6 - \frac{1}{2}a\times6^2 = 23.52$
Subtracting: $u(6 - 4) = \frac{1}{2}a(36 - 16)$, so $u = \frac{1}{2}a(10)$
Substituting: $\frac{1}{2}a(10)(4) - \frac{1}{2}a\times4^2 = 23.52$, so $\frac{1}{2}a\times24 = 23.52$, $a = 1.96$
(a) $u = 0.98\times10 = 9.8$
(b) $9.8\sin\alpha = 1.96$, $\sin\alpha = 0.2$, $\alpha = 11.5°$

Answer: (a) 9.8 m s^{-1} (b) 11.5°

10 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix}3\\0\\1\end{pmatrix} + t\begin{pmatrix}-2\\-2\\-3\end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(1, −5, 0) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathrm{PF} = \begin{pmatrix}-2t+2\\-2t+5\\-3t+1\end{pmatrix}$

PF·d = 0 gives $17t - 17 = 0$, so $t = 1$

F = (1, −2, −2); $\mathrm{PF} = \begin{pmatrix}0\\3\\-2\end{pmatrix}$, so $|\mathrm{PF}| = \sqrt{13}$

Answer: F(1, −2, −2), distance $\sqrt{13}$