Day 153 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (2, 5, 0), (−3, 0, 4) and (−2, 0, 2).
Find angle ABC, in degrees to 1 d.p.

$\text{BA} = \begin{pmatrix} 5 \\ 5 \\ -4 \end{pmatrix}$, $\text{BC} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$

BA·BC = 13

$|\text{BA}| = \sqrt{66}$, $|\text{BC}| = \sqrt{5}$

$\cos \text{ABC} = \frac{13}{\sqrt{66} \times \sqrt{5}} = 0.7156$, so angle ABC = 44.3°

Answer: 44.3°

2 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 3 m s^{-1} and accelerates uniformly. She travels 160 m in the next 20 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $160 = 3 \times 20 + \frac{1}{2} \times a \times 20^2$, so $a = 0.5$
(b) $v = u + at = 3 + 0.5 \times 20 = 13$
(c) For the midpoint, $s = 80$: $v^2 = u^2 + 2as = 3^2 + 2 \times 0.5 \times 80$
$v^2 = 89$

Answer: (a) 0.5 m s^{-2} (b) 13 m s^{-1} (c) 9.43 m s^{-1}

3 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\sin^2 \theta - 2\cos \theta - 2 = 0$
for $0° \leq \theta \leq 360°$.

Use $\sin^2 \theta \equiv 1 - \cos^2 \theta$: $2\cos^2 \theta + 2\cos \theta = 0$
Solving the quadratic: $\cos \theta = 0$ or $\cos \theta = -1$
$\cos \theta = 0$: $\theta = 90°, 270°$
$\cos \theta = -1$: $\theta = 180°$

Answer: θ = 90°, 180°, 270°

4 Parametric equations: Cartesian form (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 4 + 4\cos \theta$, $y = -2 + 4\sin \theta$.
Find its Cartesian equation and describe the curve.

$\cos \theta = \frac{x - 4}{4}$, $\sin \theta = \frac{y + 2}{4}$

Use $\cos^2 \theta + \sin^2 \theta = 1$: $(x - 4)^2 + (y + 2)^2 = 16$

Answer: $(x - 4)^2 + (y + 2)^2 = 16$, a circle with centre (4, −2), radius 4

5 Double and triple angle values (All boards)

θ is obtuse and $\sin \theta = \frac{5}{13}$.

Find the exact values of $\sin 2\theta$, $\cos 2\theta$ and $\tan 2\theta$.

$\cos \theta = -\sqrt{1 - \left(\frac{5}{13}\right)^2} = -\frac{12}{13}$ (obtuse)

$\sin 2\theta = 2\sin \theta \cos \theta = 2 \times \frac{5}{13} \times \left(-\frac{12}{13}\right) = -\frac{120}{169}$

$\cos 2\theta = 1 - 2\sin^2 \theta = 1 - 2 \times \left(\frac{5}{13}\right)^2 = \frac{119}{169}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = -\frac{120}{119}$

Answer: $\sin 2\theta = -\frac{120}{169}$, $\cos 2\theta = \frac{119}{169}$, $\tan 2\theta = -\frac{120}{119}$

6 Tree diagram with conditional probability (All boards)

The tree diagram shows probabilities for events A and B.
(a) Find P(B′).
(b) Find P(A′ | B′).

(a) $P(B') = 0.7 \times 0.1 + 0.3 \times 0.7 = 0.28$

(b) $P(A' \mid B') = \dfrac{0.3 \times 0.7}{0.28}$

Answer: (a) 0.28 (b) $\frac{3}{4}$

7 Velocity, rest and total distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on the x-axis with velocity $v = (6t^2 - 48t + 90)$ m s^{-1} at time t s, $t \geq 0$, and starts at O. Find (a) the times when P is at instantaneous rest, (b) the total distance P travels in the first 7 s.

(a) $v = 6(t - 3)(t - 5) = 0$, so $t = 3$ or $t = 5$

(b) Displacement $x = \int v\,dt = 2t^3 - 24t^2 + 90t$
$x(3) = 108$, $x(5) = 100$, $x(7) = 140$
Direction changes at $t = 3$ and $t = 5$, so distance $= |108| + |100 - 108| + |140 - 100|$
Answer: (a) t = 3, t = 5 (b) 156 m

8 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{6} - 1) = 3 + \sqrt{6}$, writing x in the form $p + q\sqrt{6}$.

$x = \dfrac{3 + \sqrt{6}}{\sqrt{6} - 1}$

Multiply top and bottom by $\sqrt{6} + 1$.
Numerator: $(3 + \sqrt{6})(\sqrt{6} + 1) = 9 + 4\sqrt{6}$
Denominator: $6 - 1 = 5$
Answer: $x = \frac{9}{5} + \frac{4}{5}\sqrt{6}$

9 Recurrence relations: limits (All boards)

$u_1 = 0$ and $u_{n+1} = \frac{1}{4} u_n + 6$.

(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.001$.

(a) $L = \frac{1}{4} L + 6$ gives $L = \dfrac{6}{1 - \frac{1}{4}} = 8$

(b) $u_n - L = (u_1 - L)\, r^{n-1} = (-8) \times \left(\frac{1}{4}\right)^{n-1}$

$|-8| \times \left(\frac{1}{4}\right)^{n-1} < \dfrac{1}{1000}$

Check: $n = 7$ gives 0.00195, $n = 8$ gives 0.00049
Answer: (a) L = 8 (b) n = 8

10 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the length (mm), is normally distributed with mean 80 and standard deviation 2.
Find (a) P(X > 77) (b) P(77 < X < 78), each to 3 s.f.

(a) $z = \dfrac{77 - 80}{2} = -1.50$, $P(Z > -1.50) = 0.933$

(b) $z_1 = -1.50$, $z_2 = \dfrac{78 - 80}{2} = -1.00$

$P(77 < X < 78) = \Phi(-1.00) - \Phi(-1.50) = 0.0918$
Answer: (a) 0.933 (b) 0.0918