Day 152 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean visibility (Dm) in the same month are:
Camborne: mean 13.5, standard deviation 1.8;
Leeming: mean 16.5, standard deviation 1.7.
Make two comparisons.

Means: Leeming has the higher mean (16.5 against 13.5), so on average the mean visibility is greater at Leeming
Standard deviations: Camborne is larger at Camborne (1.8 against 1.7)
so the readings at Camborne are more variable and those at Leeming are more consistent
Answer: Leeming higher on average; Camborne more variable

2 Double angle equations (All boards)

Solve $\cos 2\theta + \sin\theta = 1$
for $0° \le \theta \le 360°$.

$\cos 2\theta \equiv 1 - 2\sin^2\theta$: $2\sin^2\theta - \sin\theta = 0$

Solve the quadratic: $\sin\theta = \frac{1}{2}$ or $\sin\theta = 0$

$\sin\theta = \frac{1}{2}$: $\theta = 30°, 150°$

$\sin\theta = 0$: $\theta = 0°, 180°, 360°$
Answer: θ = 0°, 30°, 150°, 180°, 360°

3 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 4.9 m s^{-1} from a point h m above horizontal ground, and hits the ground 3 s later.
Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: $s = -h$, $u = 4.9$, $a = -9.8$, $t = 3$
$-h = 4.9\times3 - 4.9\times3^2$, so $h = 29.4$
(b) At the highest point $v = 0$: $0 = 4.9^2 - 2\times9.8\times s$
$s = 1.225$ m above the point of projection
Greatest height above the ground = 29.4 + 1.225
Answer: (a) 29.4 m (b) 30.625 m

4 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^3 (3x + 2)^2\,dx$.

Integrate: $[\frac{(3x + 2)^3}{9}]$

Substitute the limits and subtract: 147
Answer: 147

5 Hypothesis test for a normal mean (All boards)

The mean fill volume of bottles (ml) is normally distributed with standard deviation 4; it was previously 89.
A random sample of 64 has mean 89.3.
Test, at the 10% level, whether the mean has increased.

H_0: $\mu = 89$, H_1: $\mu > 89$

$\bar{X} \sim N(89, \frac{4^2}{64})$, $z = \frac{89.3 - 89}{\frac{4}{8}} = 0.600$

p-value = 0.274 > 0.1
Do not reject H_0: no evidence that the mean has increased.
Answer: z = 0.600, p = 0.274; do not reject H_0

6 Foot of a perpendicular (All boards)

The line l has equation $x - y = 5$.
(a) Find the coordinates of the foot of the perpendicular from P(6, −1) to l.
(b) Hence find the shortest distance from P to l, in exact form.

Gradient of l = 1, so perpendicular gradient = −1
Perpendicular through P: $-x - y = -5$
Solve with l: x = 5, y = 0
Distance$^2 = (6 - 5)^2 + (-1 - 0)^2 = 2$
Answer: (a) (5, 0) (b) $\sqrt{2}$

7 Greatest and least values using R-form (All boards)

$g(\theta) = \frac{1}{19 + 15\sin\theta + 8\cos\theta}$
(a) Write $15\sin\theta + 8\cos\theta$ in the form $R\sin(\theta + \alpha)$, with α acute.
(b) Find the greatest and least values of $g(\theta)$ and the smallest positive θ at which the greatest value occurs (1 d.p.).

$R = \sqrt{15^2 + 8^2} = 17$, $\alpha = \tan^{-1}\frac{8}{15} = 28.1°$
Denominator = $19 + 17\sin(\theta + 28.1°)$, ranging from 2 to 36
Greatest g = $\frac{1}{2}$ (denominator least), least g = $\frac{1}{36}$
Least denominator when $\sin(\theta + 28.1°) = -1$: $\theta + 28.1° = 270°$
$\theta = 241.9°$
Answer: R = 17, α = 28.1°; greatest $\frac{1}{2}$, least $\frac{1}{36}$; θ = 241.9°

8 Recurrence relations: limits (All boards)

$u_1 = -6$ and $u_{n+1} = \frac{1}{3}u_n + 4$.
(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.1$.

(a) $L = \frac{1}{3}L + 4$ gives $L = \frac{4}{1 - \frac{1}{3}} = 6$

(b) $u_n - L = (u_1 - L)\,r^{n-1} = (-12) \times \left(\frac{1}{3}\right)^{n-1}$

$|-12| \times \left(\frac{1}{3}\right)^{n-1} < \frac{1}{10}$
Check: n = 5 gives 0.14815, n = 6 gives 0.04938
Answer: (a) L = 6 (b) n = 6

9 Velocity, rest and total distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on the x-axis with velocity $v = (6t^2 - 36t + 48)$ m s^{-1} at time t s, $t \geq 0$, and starts at O. Find (a) the times when P is at instantaneous rest, (b) the total distance P travels in the first 7 s.

(a) $v = 6(t - 2)(t - 4) = 0$, so t = 2 or t = 4
(b) Displacement $x = \int v\,dt = 2t^3 - 18t^2 + 48t$
x(2) = 40, x(4) = 32, x(7) = 140
Direction changes at t = 2 and t = 4, so distance = $|40| + |32 - 40| + |140 - 32|$
Answer: (a) t = 2, t = 4 (b) 156 m

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10 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^{-1} in the expansion of

$(2x^3 - \frac{1}{x^2})^8$.

General term: ${}^8C_r\,(2x^3)^{8-r}\,(-x^{-2})^r$

Power of x: $3(8 - r) - 2r = -1$

$24 - 5r = -1$ so $r = 5$

Coefficient = ${}^8C_5 \times 2^3 \times (-1)^5 = -448$

Answer: -448