**Day 151** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Smooth incline held by a string (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg particle rests on a smooth plane inclined at $\alpha$, $\tan\alpha = \frac{3}{4}$, held by a string in the vertical plane through a line of greatest slope, at angle $\theta$ above the slope, $\tan\theta = \frac{3}{4}$. Find the tension and the normal reaction. Take g = 9.8.

$\sin\alpha = 0.6$, $\cos\alpha = 0.8$, $\sin\theta = 0.6$, $\cos\theta = 0.8$
Along the plane: $T\cos\theta = 4g\sin\alpha$, so $T = 29.4$
Perpendicular: $R + T\sin\theta = 4g\cos\alpha$
$R = 4g\cos\alpha - T\sin\theta = 31.4 - 29.4\times0.600$
**Answer: T = 29.4 N, R = 13.72 N**

## 2 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x - 1$ and $g(x) = x^2 - 4$.
Solve $gf(x) = 0$.

$gf(x) = g(2x - 1) = (2x - 1)^2 - 4$
$(2x - 1)^2 - 4 = 0$
$2x - 1 = \pm2$
**Answer: $x = -\frac{1}{2}$ or $x = \frac{3}{2}$**

## 3 Proving trigonometric identities (All boards)

Prove the identity
$(\cos A + \sin A)^2 \equiv 1 + \sin 2A$

$LHS = \cos^2 A + 2\sin A\cos A + \sin^2 A$
$= (\cos^2 A + \sin^2 A) + 2\sin A\cos A = 1 + \sin 2A = RHS$
**Answer: Proved**

## 4 Critical values for a binomial distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(36, 0.5)$.
Find the largest value of c such that $P(X \le c) < 0.01$.

$P(X \le 10) = 0.00567$
$P(X \le 11) = 0.0144$
$0.00567 < 0.01$ but $0.0144 > 0.01$
**Answer: c = 10**

## 5 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 3 kg is acted on by forces $F_1 = (9\mathbf{i} - \mathbf{j})$ N and $F_2$. Its acceleration is $(-2\mathbf{i} - 3\mathbf{j})$ m s$^{-2}$. Find (a) $F_2$, (b) the angle between $F_2$ and $\mathbf{i}$, to 1 d.p.

$F = ma$: $F_1 + F_2 = 3(-2\mathbf{i} - 3\mathbf{j}) = -6\mathbf{i} - 9\mathbf{j}$
(a) $F_2 = (-6\mathbf{i} - 9\mathbf{j}) - (9\mathbf{i} - \mathbf{j}) = -15\mathbf{i} - 8\mathbf{j}$
(b) $|F_2| = \sqrt{(-15^2 + -8^2)} = 17$
$\cos\theta = -15 \div 17$, so $\theta = 151.9°$
**Answer: (a) $F_2 = (-15\mathbf{i} - 8\mathbf{j})$ N (b) 151.9°**

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## 6 Collinear points and ratios (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A, B and C have position vectors $\begin{pmatrix}-1\\1\\-2\end{pmatrix}$, $\begin{pmatrix}-1\\4\\0\end{pmatrix}$ and $\begin{pmatrix}-1\\10\\k\end{pmatrix}$ relative to O, where k is a constant. Given that A, B and C are collinear, find k and the ratio AB : BC.

$\text{AB} = \begin{pmatrix}0\\3\\2\end{pmatrix}$, and $\text{AC} = \begin{pmatrix}0\\9\\k+2\end{pmatrix}$

AC = λAB: component 3 gives $6 = 2\lambda$, so $\lambda = 3$

Hence $k = -2 + 3 \times 2 = 4$

AC = 3 AB, so BC = 2 AB: AB : BC = 1 : 2

**Answer: k = 4, AB : BC = 1 : 2**

## 7 Range of a composite function (All boards)

$f(x) = x^2 - 6x + 13$, $x \in \mathbb{R}$, and $g(x) = \frac{1}{x+1}$, $x > -1$.

(a) Find the range of f.

(b) Show that gf is defined and find its range.

$f(x) = (x-3)^2 + 4 \geq 4$

So $f(x) \geq 4 > -1$, which lies in the domain of g, so gf is defined.

$\text{gf}(x) = \frac{1}{f(x)+1}$, with $f(x) \geq 4$, so $f(x) + 1 \geq 5$

Largest value is $\frac{1}{5} = \frac{1}{5}$ at the minimum of f; $\text{gf}(x) > 0$.

**Answer: (a) $f(x) \geq 4$ (b) $0 < \text{gf}(x) \leq \frac{1}{5}$**

## 8 Equations with cosec x and cot x (All boards)

Solve $\text{cosec}^2 x + \cot x - 3 = 0$ for $0° < x < 360°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $\cot^2 x + \cot x - 2 = 0$

$\cot x = 1$ or $\cot x = -2$

$\cot x = 1$ means $\tan x = 1$: $x = 45.0°, 225.0°$

$\cot x = -2$ means $\tan x = -\frac{1}{2}$: $x = 153.4°, 333.4°$

**Answer: x = 45.0°, 153.4°, 225.0°, 333.4°**

## 9 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2t^2 - 2$, $y = -t(t^2 - 16)$.

Find the coordinates of the points where the curve meets the x-axis.

y = 0: t = 0 or $t^2 = 16$, so $t = 0, 4, -4$

t = 0: x = −2

t = ±4: x = 30

**Answer: (−2, 0) and (30, 0)**

## 10 Conditional normal probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(55, 4^2)$. Find $P(X > 59 \mid X > 55)$ to 3 significant figures.

Since X > 59 implies X > 55: $P = \frac{P(X > 59)}{P(X > 55)}$

P(X > 59) = P(Z > 1.00) = 0.159

P(X > 55) = P(Z > 0) = 0.500

$P = \frac{0.159}{0.500} = 0.317$

**Answer: 0.317**