Day 150 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2 x - 8\sin x - 8 = 0$
for $-180° \le x \le 180°$.

Use $\cos^2 x \equiv 1 - \sin^2 x$: $2\sin^2 x + 8\sin x + 6 = 0$
Solving the quadratic: $\sin x = -1$ or $\sin x = -3$
$\sin x = -1$: $x = -90°$
$\sin x = -3$ has no solutions ($|\sin x| \le 1$)
Answer: $x = -90°$

2 Independent and mutually exclusive events (All boards)

$P(A) = 0.8$ and $P(A \cup B) = 0.82$.
(a) Given that A and B are independent, find P(B).
(b) Given instead that A and B are mutually exclusive, find P(B).
(c) In the case in (a), find $P(A' \cap B)$.

(a) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, so $0.82 = 0.8 + P(B)(1 - 0.8)$
$P(B) = \frac{0.82 - 0.8}{0.2} = 0.1$
(b) $P(A \cap B) = 0$, so $P(B) = 0.82 - 0.8 = 0.02$
(c) $P(A' \cap B) = P(A')P(B) = 0.2 \times 0.1 = 0.02$
Answer: (a) 0.1 (b) 0.02 (c) 0.02

3 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(−6, 3) and B(14, 3) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(4, −7) lies on the circle.

Centre = midpoint of AB = (4, 3)
$r^2 = (-10)^2 + (0)^2 = 100$
$(x - 4)^2 + (y - 3)^2 = 100$: $x^2 + y^2 - 8x - 6y - 75 = 0$
(b) $CM^2 = (0)^2 + (-10)^2 = 100 = r^2$
Answer: (a) $x^2 + y^2 - 8x - 6y - 75 = 0$

4 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 12 kg box is pulled from rest along rough horizontal ground by a horizontal force of 56.64 N and moves 10 m in 5 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

$s = \frac{1}{2}at^2$: $10 = \frac{1}{2}a \times 5^2$, so $a = 0.8$
(a) $P - \mu 12\,g = 12\,a$: $56.64 - 117.6\,\mu = 9.6$, so $\mu = 0.4$
(b) $v = at = 0.8 \times 5 = 4$; after removal deceleration $= \mu g = 3.92$
$0 = 4^2 - 2 \times 3.92 \times s$, $s = 2.04$
Answer: (a) $\mu = 0.4$ (b) 2.04 m

5 Arithmetic series: S_n formula (All boards)

The sum of the first n terms of an arithmetic series is $S_n = 4n^2 + n$.
(a) Find the first term and the common difference.
(b) Find u_{23}.

$u_1 = S_1 = 5$
$S_2 = 18$, so $u_2 = 13$
$d = u_2 - u_1 = 8$
$u_{23} = 5 + 22 \times 8$
Answer: (a) $a = 5$, $d = 8$ (b) 181

6 Exponential growth and decay modelling (All boards)

A radioactive sample decays so that its mass is $M = M_0e^{-kt}$ grams, with t in days.

Its half-life is 5 days. Show that $k = \frac{\ln 2}{5}$ and find how long it takes for 99% of the sample to decay.

Half-life: $e^{-5k} = \frac{1}{2}$, so $5k = \ln 2$ and $k = \frac{\ln 2}{5}$

99% decayed means 1% remains: $e^{-kt} = 0.01$

$kt = -\ln 0.01 = \ln 100$, so $t = \frac{5 \ln 100}{\ln 2} = 33.2$ days

Answer: t = 33.2 days

7 Conditional binomial probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(16, 0.4). Find $P(X = 3 \mid X \geq 2)$ to 3 significant figures.

$P(X = 3 \mid X \geq 2) = \frac{P(X = 3)}{P(X \geq 2)}$ (since X = 3 implies $X \geq 2$)

$P(X = 3) = 0.0468$

$P(X \geq 2) = 1 - P(X \leq 1) = 0.997$

Ratio = 0.0470

Answer: 0.0470

8 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against ln x for $y = A x^n$.
Find the value of n and the exact value of A.

$\ln y = \ln A + n \ln x$: gradient n, intercept ln A

$n = \frac{12 - 6}{3 - 1} = 3$

At (1, 6): $\ln A = 6 - 3 \times 1 = 3$, so $A = e^3$

Answer: n = 3, A = e^3

9 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle α above the horizontal, $\tan \alpha = \frac{12}{5}$, and passes through A, 30 m horizontally from O and 27.9 m above the ground. Find the speed of projection. Take g = 9.8.

Let the speed be U. Horizontal: $30 = U \cos \alpha \times t$, so $t = 30 \div (\frac{5}{13} U)$

Vertical: $27.9 = U \sin \alpha \times t - 4.9t^2$

Eliminate t: $y = x \tan \alpha - 4.9x^2 \div (U^2 \cos^2 \alpha)$

$27.9 = 72 - 4.9 \times 30^2 \div (\frac{25}{169} U^2)$

$U^2 = 29811.6 \div 44.1 = 676$, U = 26

Answer: 26 m s^{-1}

10 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point (8, −1). Its centre lies in the fourth quadrant.
Find the equations of the two possible circles.

Centre (r, −r), radius r > 0.

Through (8, −1): $(8 - r)^2 + (-1 + r)^2 = r^2$

$r^2 - 18r + 65 = 0$

$(r - 5)(r - 13) = 0$, so r = 5 or 13

Answer: $(x - 5)^2 + (y + 5)^2 = 25$ or $(x - 13)^2 + (y + 13)^2 = 169$