Day 15 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A stratified sample of 30 students is taken from three groups.
5 of the sample are from Arts, which has 40 students.
(a) Find the total number of students.
(b) Science has 96 students. How many are sampled?

Arts sampled fraction = $\frac{5}{40} = \frac{1}{8}$

Total = $30 \div \frac{1}{8} = 240$

Science: $96 \times \frac{1}{8} = 12$

Answer: (a) 240 (b) 12

2 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 1430 m. Find (a) V, (b) the deceleration in the final stage. Give answers to 3 s.f. if not exact.

(a) Distance = area of trapezium = $\frac{1}{2}(80 + 50) \times V$
$1430 = 65\,V$, so $V = 22$
(b) Deceleration = $V \div 15 = 22 \div 15$
Answer: (a) V = 22 (b) 1.47 m s^{-2}

3 Logarithmic equations (All boards)

Solve $2\log_2 x - \log_2(x + 48) = 2$.

$\log_2 \frac{x^2}{x + 48} = 2$, so $\frac{x^2}{x + 48} = 2^2 = 4$
$x^2 = 4(x + 48)$: $x^2 - 4x - 192 = 0$
$x = 16$ or $x = -12$; $\log_2 x$ needs $x > 0$, so reject the negative root
Answer: x = 16

4 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Show by counter-example that the statement "If ab is a multiple of 6, then a or b is a multiple of 6" is false.

Take a = 2 and b = 3:
ab = 6 is a multiple of 6,
but neither 2 nor 3 is a multiple of 6.
Answer: a = 2, b = 3

5 Equations with sin 2x and a trig function (All boards)

Solve $\sin 2x = -\sqrt{2} \sin x$ for $0° \leq x \leq 360°$.

$\sin 2x \equiv 2\sin x \cos x$: $\sin x (2\cos x + \sqrt{2}) = 0$
$\sin x = 0$: $x = 0°, 180°, 360°$

$\cos x = \frac{-\sqrt{2}}{2}$: $x = 135°, 225°$

Do not divide by sin x, as that would lose solutions
Answer: x = 0°, 135°, 180°, 225°, 360°

6 Surds: sums of fractions (AQA, Edexcel, OCR, MEI, Eduqas)

Show that $\frac{2}{\sqrt{5} - 2} + \frac{4}{\sqrt{5} + 2}$ can be written as $p + q\sqrt{5}$, and find p and q.

Common denominator $(\sqrt{5} - 2)(\sqrt{5} + 2) = 5 - 4 = 1$
Numerator = $2(\sqrt{5} + 2) + 4(\sqrt{5} - 2)$
$= -4 + 6\sqrt{5}$
Divide by 1.
Answer: p = −4, q = 6

7 Acceleration function of time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on the x-axis. At time t s its acceleration is $(12t - 12)$ m s^{-2}. When t = 0 it is at O with velocity 10 m s^{-1}. Find (a) its velocity when the acceleration is zero, (b) its displacement from O then.

$a = 0$ when $t = 1$

$v = \int a\,dt = 6t^2 - 12t + 10$ (using $v = 10$ when $t = 0$)

$v(1) = 4$

$x = \int v\,dt = 2t^3 - 6t^2 + 10t$ ($x = 0$ when $t = 0$)

$x(1) = 6$

Answer: (a) 4 m s^{-1} (b) 6 m

8 Exponential growth and decay modelling (All boards)

A car costs £12000 new. Its value falls by 10% each year, so after t years its value is £V. (a) Write down V in terms of t. (b) Find, to 3 s.f., the time when the value is £6000.

$V = 12000 \times 0.9^t$

$12000 \times 0.9^t = 6000$ gives $0.9^t = \frac{1}{2}$

$t = \frac{\ln(1}{2)/\ln 0.9} = 6.58$ years

Answer: (a) $V = 12000 \times 0.9^t$ (b) $t = 6.58$ years

9 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Eggs are defective with probability 0.2, independently. A tray of 9 eggs is rejected if it has at least 2 defective. From 6 such trays, find the probability that exactly 3 are rejected (3 s.f.).

$Y \sim B(9, 0.2)$. $P(\text{rejected}) = P(Y \geq 2) = 1 - 0.436 = 0.564$

$W \sim B(6, 0.564)$ is the number rejected

$P(W = 3) = C(6, 3)(0.564)^3(1 - 0.564)^3 = 0.297$

Answer: 0.297

10 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^{-15} in the expansion of

$(3x - \frac{1}{x^2})^9$.

General term: ${}^9C_r\,(3x^1)^{9-r}\,(-x^{-2})^r$

Power of x: $1(9 - r) - 2r = -15$

$9 - 3r = -15$ so $r = 8$

Coefficient $= {}^9C_8 \times 3^1 \times (-1)^8 = 27$

Answer: 27