Day 149 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Inverse of a rational function (All boards)

$f(x) = \frac{-5x+1}{2x+2}$, $x \in \mathbb{R}$, $x \neq -1$.

Find $f^{-1}(x)$ and state its domain.

$y = \frac{-5x+1}{2x+2}$: $y(2x+2) = -5x+1$

$2xy + 2y = -5x + 1$

$x(2y+5) = -2y + 1$

$x = \frac{-2y+1}{2y+5}$, then swap x and y

Answer: $f^{-1}(x) = \frac{-2x+1}{2x+5}$, $x \neq -\frac{5}{2}$

2 Quotient rule: stationary points (All boards)

Find the coordinates of the stationary points of

$y = \frac{x^2}{x-5}$.

$\frac{dy}{dx} = \frac{(x-5) \times 2x - x^2 \times 1}{(x-5)^2} = \frac{x^2 - 10x}{(x-5)^2} = \frac{x(x-10)}{(x-5)^2}$

$\frac{dy}{dx} = 0$ when $x = 0$ or $x = 10$

$x = 0$: $y = 0$; $x = 10$: $y = \frac{100}{5} = 20$

Answer: (0, 0), (10, 20)

3 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the length (mm), is normally distributed with mean 77 and standard deviation 2.
Find (a) $P(X > 81.8)$ (b) $P(81.8 < X < 82.8)$, each to 3 s.f.

(a) $z = \frac{81.8 - 77}{2} = 2.40$, $P(Z > 2.40) = 0.00820$

(b) $z_1 = 2.40$, $z_2 = \frac{82.8 - 77}{2} = 2.90$

$P(81.8 < X < 82.8) = \Phi(2.90) - \Phi(2.40) = 0.00633$

Answer: (a) 0.00820 (b) 0.00633

4 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 19.6 m s^{-1} from a point h m above horizontal ground, and hits the ground 6 s later.
Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: $s = -h$, $u = 19.6$, $a = -9.8$, $t = 6$

$-h = 19.6 \times 6 - 4.9 \times 6^2$, so $h = 58.8$

(b) At the highest point $v = 0$: $0 = 19.6^2 - 2 \times 9.8 \times s$

$s = 19.6$ m above the point of projection

Greatest height above the ground = 58.8 + 19.6

Answer: (a) 58.8 m (b) 78.4 m

5 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value of $\sin(\arcsin\frac{3}{5} + \arcsin\frac{3}{5})$.

Let $A = \arcsin\frac{3}{5}$ and $B = \arcsin\frac{3}{5}$, both acute

$\cos A = \frac{4}{5}$, $\cos B = \frac{4}{5}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B = \frac{3}{5} \times \frac{4}{5} + \frac{4}{5} \times \frac{3}{5} = \frac{24}{25}$

Answer: $\frac{24}{25}$

6 Foot of a perpendicular (All boards)

The line l has equation $3x - y = 2$.
(a) Find the coordinates of the foot of the perpendicular from P$(-1, -3)$ to l.
(b) Hence find the shortest distance from P to l, in exact form.

Gradient of l = 3, so perpendicular gradient = $-\frac{1}{3}$

Perpendicular through P: $-x - 3y = 10$

Solve with l: $x = -\frac{2}{5}$, $y = -\frac{16}{5}$

Distance^2 = $(-1 - (-\frac{2}{5}))^2 + (-3 - (-\frac{16}{5}))^2 = \frac{2}{5}$

Answer: (a) $(-\frac{2}{5}, -\frac{16}{5})$ (b) $\frac{\sqrt{10}}{5}$

7 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (4, 1, −2), (5, −2, −1) and (−1, 2, 4).
Find angle ABC, in degrees to 1 d.p.

BA = $\begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$, BC = $\begin{pmatrix} -6 \\ 4 \\ 5 \end{pmatrix}$

BA·BC = 13

|BA| = $\sqrt{11}$, |BC| = $\sqrt{77}$

cos ABC = $\frac{13}{\sqrt{11} \times \sqrt{77}}$ = 0.4467, so angle ABC = 63.5°

Answer: 63.5°

8 Outliers (AQA, Edexcel, OCR, MEI, Eduqas)

A data set has mean 71 and standard deviation 4.
A value is treated as an outlier if it is more than 3 standard deviations from the mean.
Which of 56, 62, 70, 92 are outliers?

3 standard deviations = 3 × 4 = 12
Lower limit = 71 − 12 = 59
Upper limit = 71 + 12 = 83
Outliers are values below 59 or above 83
Answer: 56, 92

9 Circle through two points with centre on a line (AQA, Edexcel, OCR, MEI, Eduqas)

A circle passes through A$(-4, -11)$ and B$(2, 7)$.
Its centre lies on the line $y = -3$.
Find the equation of the circle.

Centre $(h, -3)$ is equidistant from A and B.
$(h + 4)^2 + (8)^2 = (h - 2)^2 + (-10)^2$
Solving gives h = 2, so centre $(2, -3)$
$r^2 = (-6)^2 + (-8)^2 = 100$
Answer: $(x - 2)^2 + (y + 3)^2 = 100$

10 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (5 m, 20 kg) rests with A on rough ground, $\mu = 0.2$, and B against a smooth wall, B being 3 m above the ground. A man of mass 70 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(5^2 - 3^2)} = 4$ m from the wall

Vertical: $R = (20 + 70)g$. Limiting friction: $F = \mu R = 0.2(20 + 70)g$

Horizontal: $S = F$ (wall reaction S)

Moments about A (man at distance x along ladder, horizontal distance $x \times 4/5$):

$S \times 3 = 20\,g \times 2 + 70\,g \times (4x \div 5)$

$18 \times 3 = 40 + 56x$ (g cancels), so $x = 0.25$

Answer: 0.25 m