Day 148 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Index equations solved with logarithms (All boards)

Solve $3^{4x+3} = 2^{x+1}$.
Give your answer to 3 significant figures.

Take logs: $(4x + 3) \ln 3 = (x + 1) \ln 2$
Collect x terms: $x(4 \ln 3 - \ln 2) = \ln 2 - 3 \ln 3$
$x = \dfrac{\ln 2 - 3 \ln 3}{4 \ln 3 - \ln 2}$
$x = -0.703$
Answer: $x = -0.703$

2 Transformations of a turning point (All boards)

The curve $y = f(x)$ has a maximum point at (2, 5).
Find the coordinates of the corresponding point on:
(a) $y = f(-x)$
(b) $y = f(x + 4)$
(c) $y = f(x) + 5$

(a) $y = f(-x)$: $(-2, 5)$
(b) $y = f(x + 4)$: $(-2, 5)$
(c) $y = f(x) + 5$: $(2, 10)$
A stretch or translation moves the point in the stated direction only.
Answer: (a) $(-2, 5)$; (b) $(-2, 5)$; (c) $(2, 10)$

3 Beam on two supports (AQA, Edexcel, OCR, MEI)

A uniform beam AB rests on supports at C and D, with a particle attached at B. Find the reactions at C and D. Take $g = 9.8$.

Moments about C: $R_D \times 7 = 30\,g \times 3.5 + 10\,g \times 8.5$
$R_D = 266$
Resolving vertically: $R_C + R_D = 40\,g = 392$
$R_C = 392 - 266$
Answer: $R_C = 126$ N, $R_D = 266$ N

4 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample is taken from a numbered list of 200 items.
The 6th item chosen is number 28 and the 7th item chosen is number 33.
Find the sampling interval, the number of the first item chosen and the size of the sample.

Interval $= (33 - 28) \div (7 - 6) = 5$
First item $= 28 - 5 \times 5 = 3$
Sample size $= 200 \div 5 = 40$
Answer: Interval 5, first 3, size 40

5 Range of a composite function (All boards)

$f(x) = x^2 + 4x + 6$, $x \in \mathbb{R}$, and $g(x) = \dfrac{10}{x + 3}$, $x > -3$.

(a) Find the range of f.
(b) Show that gf is defined and find its range.

$f(x) = (x + 2)^2 + 2 \geq 2$
So $f(x) \geq 2 > -3$, which lies in the domain of g, so gf is defined.
$gf(x) = \dfrac{10}{f(x) + 3}$, with $f(x) \geq 2$, so $f(x) + 3 \geq 5$
Largest value is $\dfrac{10}{5} = 2$ at the minimum of f; $gf(x) > 0$.
Answer: (a) $f(x) \geq 2$ (b) $0 < gf(x) \leq 2$

6 Volumes of revolution (CIE (9709); Further Maths on other boards)

The region bounded by the curve $y = 2\sqrt{x}$, the x-axis, $x = 0$ and $x = 2$ is rotated through 360° about the x-axis. Find the exact volume of the solid formed.

$y^2 = 4x$, so $V = \pi\int_0^2 y^2\,dx$

$= \pi \times 8$

$V = 8\pi$

Answer: 8π

7 Critical values for a binomial distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(28, 0.2)$.

Find the largest value of c such that $P(X \leq c) < 0.1$.

$P(X \leq 2) = 0.0612$

$P(X \leq 3) = 0.160$

$0.0612 < 0.1$ but $0.160 > 0.1$

Answer: c = 2

8 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(27 + 3x)^{\frac{1}{3}}$ in ascending powers of x, up to and including x^2.

State the range of x for which the expansion is valid.

$(27 + 3x)^{\frac{1}{3}} = 27^{\frac{1}{3}}(1 + \frac{3x}{27})^{\frac{1}{3}}$

$27^{\frac{1}{3}} = 3$

Expand $(1 + y)^{\frac{1}{3}}$ with $y = \frac{3x}{27}$

Valid for $|\frac{3x}{27}| < 1$, i.e. $|x| < 9$

Answer: $3 + \frac{1}{9}x - \frac{1}{243}x^2$; $|x| < 9$

9 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\sin^2\theta - 7\cos\theta - 5 = 0$

for $0 \leq \theta \leq 2\pi$.

Use $\sin^2\theta \equiv 1 - \cos^2\theta$: $2\cos^2\theta + 7\cos\theta + 3 = 0$

Solving the quadratic: $\cos\theta = -\frac{1}{2}$ or $\cos\theta = -3$

$\cos\theta = -\frac{1}{2}$: $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\cos\theta = -3$ has no solutions ($|\cos\theta| \leq 1$)

Answer: $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

10 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 8 kg particle hangs in equilibrium from two light strings, as shown. Find the tension in each string (T_1 in the string at angle α). Take $g = 9.8$.

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$, $\sin\beta = 0.6$, $\cos\beta = 0.8$

Horizontal: $T_1\cos\alpha = T_2\cos\beta$, so $T_2 = \frac{15}{13}T_1$

Vertical: $T_1\sin\alpha + T_2\sin\beta = 8g = 78.4$

$T_1(\frac{5}{13} + \frac{9}{13}) = 78.4$

$T_1 = 72.8$, $T_2 = 84.0$

Answer: $T_1 = 72.8$ N, $T_2 = 84$ N