Day 145 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 2nd term of an arithmetic sequence is −1 and the 6th term is 15.
Find the first term and common difference, and the sum of the first 30 terms.

$u_6 - u_2 = 4\ d = 15 - (-1)$ so $d = 4$
$a + 1\ d = -1$ so $a = -5$
$S_{30} = \frac{30}{2}(2 \times (-5) + 29 \times 4)$
Answer: $a = -5$, $d = 4$, $S_{30} = 1590$

2 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample is taken from a numbered list of 500 items.
The 5th item chosen is number 98 and the 6th item chosen is number 118.
Find the sampling interval, the number of the first item chosen and the size of the sample.

Interval = (118 − 98) ÷ (6 − 5) = 20
First item = 98 − 4 × 20 = 18
Sample size = 500 ÷ 20 = 25
Answer: Interval 20, first 18, size 25

3 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 34.3 m s^{-1} from a point h m above horizontal ground, and hits the ground 10 s later.
Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: $s = -h$, $u = 34.3$, $a = -9.8$, $t = 10$
$-h = 34.3 \times 10 - 4.9 \times 10^2$, so $h = 147$
(b) At the highest point $v = 0$: $0 = 34.3^2 - 2 \times 9.8 \times s$
$s = 60.025$ m above the point of projection
Greatest height above the ground = 147 + 60.025
Answer: (a) 147 m (b) 207.025 m

4 Equation after a sequence of transformations (All boards)

The curve with equation $y = 2x^2 - 5x + 4$ is transformed by a reflection in the x-axis, followed by a stretch, scale factor 2, parallel to the y-axis.
Find the equation of the image curve.

Reflect in x-axis: y becomes −y: $y = -2x^2 + 5x - 4$
Stretch parallel to y: multiply by 2: $y = -4x^2 + 10x - 8$
Answer: $y = -4x^2 + 10x - 8$

5 Conservation of momentum (CIE)

Particles A (8 kg) and B (4 kg) move towards each other on a smooth horizontal line at 8 m s^{-1} and 7 m s^{-1}. They collide and coalesce. Find (a) the speed of the combined particle, (b) the kinetic energy lost in the collision.

Conservation of momentum (A's direction positive):
$8 \times 8 - 4 \times 7 = 12\ v$, so $v = 3$
KE before = $\frac{1}{2} \times 8 \times 8^2 + \frac{1}{2} \times 4 \times 7^2 = 354$
KE after = $\frac{1}{2} \times 12 \times 3^2 = 54$
Loss = 354 − 54
Answer: (a) 3 m s^{-1} (b) 300 J

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6 Equations with cosec x and cot x (All boards)

Solve $4\text{cosec}^2 x + 6\cot x - 22 = 0$
for $0° < x < 360°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $4\cot^2 x + 6\cot x - 18 = 0$

$\cot x = \frac{3}{2}$ or $\cot x = -3$

$\cot x = \frac{3}{2}$ means $\tan x = \frac{2}{3}$: $x = 33.7°, 213.7°$

$\cot x = -3$ means $\tan x = -\frac{1}{3}$: $x = 161.6°, 341.6°$

Answer: x = 33.7°, 161.6°, 213.7°, 341.6°

7 Normal approximation to the Poisson (CIE)

$X \sim \text{Po}(20)$. Use a suitable approximation to find $P(X \leq 27)$, to 3 significant figures.

$\lambda > 15$, so $Y \sim N(20, 20)$ approximately
Continuity correction: $P(Y < 27.5)$
$z = 1.68$, so probability = 0.953
Answer: 0.953

8 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $x^2 + 2xy + 2y^2 = 1$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{2x + 2y}{2x + 4y}$

Horizontal tangent: $2x + 2y = 0$, so $y = -x$

Substitute: $x^2 = 1$, so $x^2 = 1$ and $x = \pm 1$

Points: $(1, -1)$ and $(-1, 1)$

Answer: (1, −1) and (−1, 1)

9 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{25x - 10}{(1 - 2x)(1 - 3x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - 2x)} + \frac{B}{(1 - 3x)}$: $A = -5$, $B = -5$

(b) $-5(1 - 2x)^{-1} = -5(1 + 2x + 4x^2 \ldots)$

Add the series for both fractions.

(c) Need $|2x| < 1$ and $|3x| < 1$

Answer: (a) $\frac{-5}{(1 - 2x)} + \frac{-5}{(1 - 3x)}$ **(b)** $-10 - 25x - 65x^2$ **(c)** $|x| < \frac{1}{3}$

10 Differential equations in context (All boards)

A drink at 90°C is left in a room at 20°C. Its temperature θ °C satisfies $\frac{d\theta}{dt} = -k(\theta - 20)$.

After 5 minutes the temperature is 60°C. Find k, then the time for it to reach 35°C, to 3 s.f.

$\int \frac{1}{\theta - 20} d\theta = \int -k\, dt$, so $\ln(\theta - 20) = -kt + C$

$t = 0$: $\theta - 20 = 70$, so $\theta - 20 = 70e^{-kt}$

$t = 5$: $40 = 70e^{-5k}$, so $k = \frac{\ln(\frac{7}{4})}{5} = 0.112$

$15 = 70e^{-kt}$: $t = \frac{\ln(\frac{14}{3})}{k} = 13.8$ minutes

Answer: k = 0.112, t = 13.8 minutes