Day 144 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{7}{25}$ and $\cos B = -\frac{3}{5}$.

Find the exact values of tan(A−B) and sin(A+B).

$\cos A = \frac{24}{25}$ (A acute), $\sin B = \frac{4}{5}$ (B obtuse), so $\tan A = \frac{7}{24}$, $\tan B = -\frac{4}{3}$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\frac{7}{24} - (-\frac{4}{3})}{1 + \frac{7}{24} \times (-\frac{4}{3})} = \frac{117}{44}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B = \frac{7}{25} \times (-\frac{3}{5}) + \frac{24}{25} \times \frac{4}{5} = \frac{3}{5}$$

Answer: $\tan(A-B) = \frac{117}{44}$, $\sin(A+B) = \frac{3}{5}$

2 Combined inequalities (All boards)

(a) Solve $4x - 1 < -6$.

(b) Solve $x^2 + 2x - 8 \geq 0$.

(c) Hence find the values of x satisfying both inequalities.

(a) $4x - 1 < -6$ so $x < -\frac{5}{4}$

(b) $(x + 4)(x - 2) \geq 0$: critical values -4 and 2

(b) gives $x \leq -4$ or $x \geq 2$

(c) Overlap of the two sets.

Answer: (a) $x < -\frac{5}{4}$ (b) $x \leq -4$ or $x \geq 2$ (c) $x \leq -4$

3 Outliers (AQA, Edexcel, OCR, MEI, Eduqas)

A data set has mean 50 and standard deviation 2.

A value is treated as an outlier if it is more than 3 standard deviations from the mean.

Which of 43, 51, 52, 65 are outliers?

3 standard deviations = 3 × 2 = 6

Lower limit = 50 − 6 = 44

Upper limit = 50 + 6 = 56

Outliers are values below 44 or above 56

Answer: 43, 65

4 Sum of multiples in a range (All boards)

Find the sum of all the integers from 1 to 66 inclusive that are not multiples of 13.

Sum of 1 to 66 = $\frac{66 \times 67}{2} = 2211$

Multiples of 13 up to 66: 13, 26, ..., 65 (5 terms)

Their sum = $\frac{5}{2}(13 + 65) = 195$

Required sum = 2211 − 195

Answer: 2016

5 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 2 kg particle rests on a rough plane inclined at α, $\tan\alpha = \frac{8}{15}$, $\mu = 0.25$. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{8}{17}$, $\cos\alpha = \frac{15}{17}$, $R = 2g\cos\alpha = \frac{294}{17}$ N

Limiting friction $= \mu R = \frac{147}{34}$ N

P smallest: friction acts up the slope: $P + \mu R = 2g\sin\alpha$, $P = \frac{784}{85} - \frac{147}{34}$

P greatest: friction acts down the slope: $P = \frac{784}{85} + \frac{147}{34}$

Answer: $4.9\text{ N} \leq P \leq 13.5\text{ N}$

6 Quotient rule: stationary points (All boards)

Find the exact coordinates of the stationary point of

$y = \frac{e^x}{x}$ for $x > 0$.

$\frac{dy}{dx} = \frac{x \times e^x - e^x \times 1}{x^2} = \frac{e^x(x-1)}{x^2}$

$e^x > 0$, so $\frac{dy}{dx} = 0$ when $x = 1$, giving $x = 1$

$y = e^1 \div 1 = e$

Answer: (1, e)

7 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that no cube number can be of the form $7k + 4$, where k is an integer.

Cubes depend only on n mod 7: n = 0, 1, 2, 3, 4, 5, 6.
Remainders of n^3 on dividing by 7: 0, 1, 1, 6, 1, 6, 6
The only possible remainders are 0, 1, 6.
The remainder 4 never occurs.
Answer: remainder 4 never occurs among cubes (mod 7)

8 Separable differential equations (All boards)

Solve the differential equation

$\frac{dy}{dx} = 6xy^2$,

given that $y = 2$ when $x = 0$. Give y in terms of x.

Separate: $\int \frac{1}{y^2}\,dy = \int 6x\,dx$, so $-\frac{1}{y} = 3x^2 + C$

$x = 0$, $y = 2$ gives $C = -\frac{1}{2}$

$-\frac{1}{y} = 3x^2 - \frac{1}{2}$

Answer: $y = \frac{2}{1 - 6x^2}$

9 Range of horizontal force on a rough slope (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg particle is in equilibrium on a rough plane inclined at α, $\tan\alpha = \frac{24}{7}$, $\mu = 0.25$. A horizontal force of P N, in the vertical plane through a line of greatest slope, pushes it towards the plane. Find the least and greatest values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = 0.96$, $\cos\alpha = 0.28$, weight $W = 39.2$ N. Normal reaction $R = W\cos\alpha + P\sin\alpha$
Least P: about to slide down, friction $F = \mu R$ up the slope
$P\cos\alpha + \mu(W\cos\alpha + P\sin\alpha) = W\sin\alpha \rightarrow P = W(\sin\alpha - \mu\cos\alpha) \div (\cos\alpha + \mu\sin\alpha) = 67.1$
Greatest P: about to slide up, friction down the slope
$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha) \rightarrow P = W(\sin\alpha + \mu\cos\alpha) \div (\cos\alpha - \mu\sin\alpha) = 1010$
Answer: least 67.1 N, greatest 1010 N

10 Arrangements with restrictions (OCR, MEI, Eduqas, CIE)

How many arrangements of the letters of NUMBERS begin and end with a consonant?

The first letter: 5 consonants
The last letter: 4 consonants left
The middle 5 letters: 5! ways
Total = $5 \times 4 \times 5! = 2400$
Answer: 2400