Day 143 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Correcting a recorded value (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

30 values have $\Sigma x = 630$ and $\Sigma x^2 = 14310$.
One value was recorded as 6 but should have been 22.
Find the correct mean and standard deviation (3 s.f.).

$\Sigma x = 630 - 6 + 22 = 646$

$\Sigma x^2 = 14310 - 6^2 + 22^2 = 14758$

Mean $= \frac{646}{30} = 21.5$

sd $= \sqrt{\frac{14758}{30} - (21.5)^2} = 5.31$

Answer: mean 21.5, sd 5.31

2 Arc length and sector area (All boards)

A sector of a circle has area 2.7 cm^2 and angle 0.6 radians.
Find the radius, the arc length and the perimeter of the sector.

Area $= \frac{1}{2}r^2\theta$: $2.7 = \frac{1}{2}r^2 \times 0.6$, so $r^2 = 9 = 3^2$

r = 3 cm; arc length $s = r\theta = 3 \times 0.6 = 1.8$ cm
Perimeter $= 2r + s = 6 + 1.8 = 7.8$ cm
Answer: r = 3 cm, arc 1.8 cm, perimeter 7.8 cm

3 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 20 kg is pushed at constant speed along rough horizontal ground, $\mu = 0.2$, by a force of P N at angle θ below the horizontal, $\tan\theta = \frac{12}{5}$. Find P and the normal reaction. Take g = 9.8. Give answers to 3 s.f. if not exact.

Constant speed, so the resultant force is zero. $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$

Vertical: $R = 20g + P\sin\theta$
Horizontal: $P\cos\theta = \mu R = 0.2(20g + P\sin\theta)$
$P(\frac{5}{13} - \frac{12}{65}) = 0.2 \times 20g$
P = 196, R = 377
Answer: P = 196 N, R = 377 N

4 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{15}{17}$ and $\cos B = -\frac{5}{13}$.
Find the exact values of tan(A+B) and sin(A−B).

$\cos A = \frac{8}{17}$ (A acute), $\sin B = \frac{12}{13}$ (B obtuse), so $\tan A = \frac{15}{8}$, $\tan B = -\frac{12}{5}$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{15}{8} + (-\frac{12}{5})}{1 - \frac{15}{8} \times (-\frac{12}{5})} = -\frac{21}{220}$$

$$\sin(A-B) = \sin A\cos B - \cos A \sin B = \frac{15}{17} \times (-\frac{5}{13}) - \frac{8}{17} \times \frac{12}{13} = -\frac{171}{221}$$

Answer: $\tan(A+B) = -\frac{21}{220}$, $\sin(A-B) = -\frac{171}{221}$

5 Regression line from summary data (AQA, Edexcel, OCR, MEI, Eduqas)

n = 10 pairs of (x, y): $\Sigma x = 151$, $\Sigma y = 248$, $\Sigma x^2 = 2841$, $\Sigma xy = 4318$.
(a) Find the equation of the regression line of y on x (3 s.f.).
(b) Estimate y when x = 19.

$S_{xx} = 2841 - \frac{151^2}{10} = 561$, $S_{xy} = 4318 - \frac{151 \times 248}{10} = 573$

$b = \frac{S_{xy}}{S_{xx}} = 1.02$

$a = \bar{y} - b\,\bar{x} = 24.8 - (1.02)(15.1) = 9.37$

(b) $y = 9.37 + (1.02)(19) = 28.8$

Answer: (a) $y = 9.37 + 1.02x$ (b) 28.8

6 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^8 in the expansion of

$(x^3 - \frac{2}{x^2})^6$.

General term: ${}^6C_r\,(x^3)^{6-r}\,(-2x^{-2})^r$

Power of x: $3(6 - r) - 2r = 8$

$18 - 5r = 8$ so $r = 2$

Coefficient = ${}^6C_2 \times 1^4 \times (-2)^2 = 60$

Answer: 60

7 Car and trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 1500 kg car tows a 500 kg trailer by a light tow-bar on a horizontal road. Driving force 1800 N; resistances 400 N (car), 200 N (trailer). Find (a) the acceleration and the tow-bar tension. At 8 m s^{-1} the tow-bar breaks. (b) How far does the trailer then travel?

Whole system: $1800 - 400 - 200 = 2000\,a$, so $a = 0.6$

Trailer: $T - 200 = 500\,a$, so $T = 200 + 500{\times}0.6 = 500$

(b) After the break the trailer decelerates at $200 \div 500 = 0.4$ m s^{-2}

$0 = 8^2 - 2{\times}0.4{\times}s$, so $s = 80$

Answer: (a) 0.6 m s^{-2}, 500 N (b) 80 m

8 Recurrence relations: limits (All boards)

$u_1 = 40$ and $u_{n+1} = \frac{1}{4}u_n + 15$.

(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.01$.

(a) $L = \frac{1}{4}L + 15$ gives $L = \frac{15}{1 - \frac{1}{4}} = 20$

(b) $u_n - L = (u_1 - L)\,r^{n-1} = 20 \times \left(\frac{1}{4}\right)^{n-1}$

$|20| \times \left(\frac{1}{4}\right)^{n-1} < \frac{1}{100}$

Check: n = 6 gives 0.01953, n = 7 gives 0.00488

Answer: (a) L = 20 (b) n = 7

9 Partial fractions: repeated factor (All boards)

Express $\frac{-2x^2 + x - 9}{(x+1)(x-1)^2}$ in the form $\frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$, and find A, B and C.

$-2x^2 + x - 9 \equiv A(x-1)^2 + B(x+1)(x-1) + C(x+1)$

$x = -1$: $-12 = A(-2)^2$ so $A = -3$

$x = 1$: $-10 = C(2)$ so $C = -5$

Coefficient of x^2: $-2 = A + B$ so $B = 1$

Answer: $A = -3$, $B = 1$, $C = -5$

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10 Double and triple angle values (All boards)

θ is obtuse and $\sin\theta = \frac{20}{29}$.

Find the exact values of $\sin 2\theta$, $\cos 2\theta$ and $\tan 2\theta$.

$\cos\theta = -\sqrt{1 - \left(\frac{20}{29}\right)^2} = -\frac{21}{29}$ (obtuse)

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{20}{29} \times \left(-\frac{21}{29}\right) = -\frac{840}{841}$

$\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2 \times \left(\frac{20}{29}\right)^2 = \frac{41}{841}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = -\frac{840}{41}$

Answer: $\sin 2\theta = -\frac{840}{841}$, $\cos 2\theta = \frac{41}{841}$, $\tan 2\theta = -\frac{840}{41}$