Day 142 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Product rule: stationary points (All boards)

$y = x^2 e^{-2x}$ for $x > 0$.

Find the exact coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 2x \times e^{-2x} + x^2 \times (-2)e^{-2x}$

$= x(2 - 2x)e^{-2x}$

$e^{-2x} > 0$ and $x > 0$, so $\frac{dy}{dx} = 0$ gives $2 - 2x = 0$ and $x = 1$

$y = e^{-2}$; $\frac{d^2y}{dx^2}$ is negative there, so it is a maximum

Answer: (1, e^{-2}), maximum

2 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

The 408 items in a list are numbered 1 to 408.

A systematic sample of 34 items is taken, starting with a random item among the first few.

The first item selected is number 10.

Find the sampling interval and the number of the last item selected.

Interval = 408 ÷ 34 = 12

Items selected: 10, 22, 34, ...

Last = 10 + (34 − 1) × 12 = 406

Answer: Interval 12, last item 406

3 Foot of a perpendicular (All boards)

The line l has equation $3x + 3y = -8$.

(a) Find the coordinates of the foot of the perpendicular from P(−1, 5) to l.

(b) Hence find the shortest distance from P to l, in exact form.

Gradient of l = −1, so perpendicular gradient = 1

Perpendicular through P: $3x - 3y = -18$

Solve with l: $x = -\frac{13}{3}$, $y = \frac{5}{3}$

Distance$^2 = (-1 - (-\frac{13}{3}))^2 + (5 - \frac{5}{3})^2 = \frac{200}{9}$

Answer: (a) $(-\frac{13}{3}, \frac{5}{3})$ (b) $\frac{10\sqrt{2}}{3}$

4 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ -1 \end{pmatrix}$ and $\begin{pmatrix} k \\ 6 \\ -5 \end{pmatrix}$ are perpendicular.

Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 + 6k + 5 = 0$

$(k + 5)(k + 1) = 0$

$k = -5$ or $k = -1$. For $k = -1$ the first vector is $\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$, with length $\sqrt{3} = \sqrt{3}$

Answer: k = −5 or −1; length $\sqrt{3}$

5 Beam on two supports (AQA, Edexcel, OCR, MEI)

A uniform beam AB rests on supports at C and D, with a particle attached at A. Find the reactions at C and D. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about C: $R_D \times 3 = 15\,g \times 1.5 - 4\,g \times 3.5$

$R_D = 27.8$

Resolving vertically: $R_C + R_D = 19\,g = 186.2$

$R_C = 186.2 - 27.8$

Answer: R_C = 158 N, R_D = 27.8 N

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6 Partial fractions: improper fraction (All boards)

Express $\frac{2x^2 - 4x - 8}{(x + 2)(x - 2)}$ in the form $2 + \frac{A}{(x + 2)} + \frac{B}{(x - 2)}$, where A and B are constants to be found.

Divide: $2x^2 - 4x - 8 = 2(x^2 - 4) + (-4x)$

So the fraction is $2 + \frac{-4x}{(x + 2)(x - 2)}$

$A(x - 2) + B(x + 2) \equiv -4x$

$x = -2$ gives $A = -2$; $x = 2$ gives $B = -2$

Answer: $A = -2$, $B = -2$

7 Regression with coded variables (AQA, Edexcel, OCR, MEI, Eduqas)

A regression line of v on w is $v = -3019 + -15w$, where $w = \frac{t - 2010}{10}$.

(a) Find the regression line of v on t in the form $v = p + qt$.

(b) Estimate v when $t = 2040$.

$v = -3019 + -15 \times \frac{t - 2010}{10}$

$= -3019 + -1.5t - -3015 = -4 + -1.5t$

(b) $v = -4 + -1.5 \times 2040 = -3064$

Answer: (a) $v = -4 - 1.5t$ (b) -3064

8 Smooth incline held by a string (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 3 kg particle rests on a smooth plane inclined at α, $\tan\alpha = \frac{5}{12}$, held by a string in the vertical plane through a line of greatest slope, at angle θ above the slope, $\tan\theta = \frac{3}{4}$. Find the tension and the normal reaction. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$, $\sin\theta = 0.6$, $\cos\theta = 0.8$

Along the plane: $T\cos\theta = 3g\sin\alpha$, so $T = 14.1$

Perpendicular: $R + T\sin\theta = 3g\cos\alpha$

$R = 3g\cos\alpha - T\sin\theta = 27.1 - 14.1 \times 0.600$

Answer: $T = 14.1$ N, $R = 18.7$ N

9 Proving trigonometric identities (All boards)

Prove the identity

$\frac{\sin 6x}{1 - \cos 6x} \equiv \cot 3x$

$\text{LHS} = \frac{2\sin 3x \cos 3x}{1 - (1 - 2\sin^2 3x)} = \frac{2\sin 3x \cos 3x}{2\sin^2 3x}$

$= \frac{\cos 3x}{\sin 3x} = \cot 3x = \text{RHS}$

Answer: Proved

10 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point (8, 1). Its centre lies in the first quadrant. Find the equations of the two possible circles.

Centre (r, r), radius $r > 0$.

Through (8, 1): $(8 - r)^2 + (1 - r)^2 = r^2$

$r^2 - 18r + 65 = 0$

$(r - 5)(r - 13) = 0$, so $r = 5$ or 13

Answer: $(x - 5)^2 + (y - 5)^2 = 25$ or $(x - 13)^2 + (y - 13)^2 = 169$