Day 141 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2 x + 3\sin x - 2 = 0$
for $0 \leq x \leq 2\pi$.

Use $\cos^2 x \equiv 1 - \sin^2 x$: $2\sin^2 x - 3\sin x = 0$

Solving the quadratic: $\sin x = 0$ or $\sin x = \frac{3}{2}$

$\sin x = 0$: $x = 0, \pi, 2\pi$

$\sin x = \frac{3}{2}$ has no solutions ($|\sin x| \leq 1$)

Answer: $x = 0, \pi, 2\pi$

2 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $x^2 - 8x + 17 > 0$ for all real values of x.

Complete the square: $x^2 - 8x + 17 = (x - 4)^2 + 1$

$(x - 4)^2 \geq 0$ for all x, so the expression is $\geq 1 > 0$.

Answer: $(x - 4)^2 + 1 \geq 1 > 0$

3 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 4.9 m s^{-1} from a point h m above horizontal ground, and hits the ground 5 s later.
Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: $s = -h$, $u = 4.9$, $a = -9.8$, $t = 5$
$-h = 4.9\times5 - 4.9\times5^2$, so $h = 98$
(b) At the highest point $v = 0$: $0 = 4.9^2 - 2\times9.8\times s$
$s = 1.225$ m above the point of projection
Greatest height above the ground = 98 + 1.225
Answer: (a) 98 m (b) 99.225 m

4 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample is taken from a numbered list of 175 items.
The 6th item chosen is number 26 and the 7th item chosen is number 31.
Find the sampling interval, the number of the first item chosen and the size of the sample.

Interval = $(31 - 26) \div (7 - 6) = 5$
First item = $26 - 5 \times 5 = 1$
Sample size = $175 \div 5 = 35$
Answer: Interval 5, first 1, size 35

5 Differentiating sec, cosec, tan and cot (All boards)

$y = \sec(2x)$.

Find $\frac{dy}{dx}$ and its exact value at $x = \frac{\pi}{6}$.

$\frac{dy}{dx} = 2\sec(2x)\tan(2x)$

At $x = \frac{\pi}{6}$ the angle $2x = \frac{\pi}{3}$, so the gradient is $4\sqrt{3}$

Answer: $\frac{dy}{dx} = 2\sec(2x)\tan(2x)$; $4\sqrt{3}$

6 Tree diagram with conditional probability (All boards)

The tree diagram shows probabilities for events R and S.
(a) Find P(S).
(b) Find P(R | S).

(a) $P(S) = 0.6 \times 0.7 + 0.4 \times 0.3 = 0.54$

(b) $P(R \mid S) = \frac{P(R \cap S)}{P(S)} = \frac{0.6 \times 0.7}{0.54}$

Answer: (a) 0.54 (b) $\frac{7}{9}$

7 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

A hemispherical bowl of radius 8 cm is filled at 60 cm^3s^{-1}. When the depth is h, $V = \pi(8h^2 - \frac{1}{3}h^3)$.

Find $\frac{dh}{dt}$ when h = 2 cm.

$\frac{dV}{dh} = \pi(16h - h^2) = \pi h(16 - h)$

$\frac{dh}{dt} = \frac{60}{\pi h(16 - h)}$

h = 2: $\frac{dh}{dt} = \frac{60}{\pi \times 2 \times 14} = \frac{15}{7\pi}$ cm s^{-1}

Answer: $\frac{15}{7\pi}$ cm/s

8 Line and curve intersection (diagram) (All boards)

The diagram shows the curve $y = x^2 + x - 2$ and the line $y = -2x - 2$, meeting at A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the midpoint of AB.

$x^2 + x - 2 = -2x - 2$: $x^2 + 3x = 0$
$(x + 3)x = 0$, $x = -3$ or 0
A(−3, 4), B(0, −2)
Midpoint: average the coordinates

Answer: (a) A(−3, 4), B(0, −2) (b) $(-\frac{3}{2}, 1)$

9 Catching up (delayed start) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes P at constant speed 10 m s^{-1}. 5 s later car B leaves P from rest, in the same direction, with acceleration 3 m s^{-2}. B overtakes A at Q. Find (a) the time B takes to reach Q, (b) PQ, (c) the speed of B at Q.

Let B take s seconds; A has then been moving for (s + 5) seconds.
B: ½×3×s^2 = 1.5 s^2 A: 10(s + 5)
1.5 s^2 − 10 s − 50 = 0 so s = 10 (s > 0)
(b) PQ = 1.5×10^2 = 150
(c) v = 0 + at = 3×10 = 30

Answer: (a) 10 s (b) 150 m (c) 30 m s^{-1}

10 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is −12 and the coefficient of x^2 is 60. Find n and k.

$nk = -12$ and $\frac{n(n-1)}{2}k^2 = 60$

$(n - 1)k = \frac{2 \times 60}{-12} = -10$ (dividing the second equation by the first)

$n\,k - (n - 1)\,k = k = -12 - (-10) = -2$

$n = \frac{-12}{-2} = 6$

Answer: n = 6, k = −2