Day 140 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = -3x + 3$ and $g(x) = x^2 + 4$.
Solve $gf(x) = 40$.

$gf(x) = g(-3x + 3) = (-3x + 3)^2 + 4$
$(-3x + 3)^2 + 4 = 40$
$-3x + 3 = \pm 6$
Answer: $x = -1$ or $x = 3$

2 Logarithmic equations (All boards)

Solve $\log_2(x + 6) + \log_2(x - 9) = 4$.

Combine: $\log_2[(x + 6)(x - 9)] = 4$, so $(x + 6)(x - 9) = 2^4 = 16$
$x^2 - 3x - 70 = 0$: $(x - 10)(x + 7) = 0$
$x = 10$ or $x = -7$; $x = -7$ makes $x - 9$ negative, so it is rejected
Answer: $x = 10$

3 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $9^{3x-2} = 3^{3x-2}$.

Write both sides as powers of 3.
$(3^2)^{3x-2} = (3^1)^{3x-2}$
$2(3x - 2) = 3x - 2$
$6x - 4 = 3x - 2$
$3x = 2$
Answer: $x = \frac{2}{3}$

4 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{24}{25}$ and $\cos B = -\frac{12}{13}$.
Find the exact values of $\tan(A+B)$ and $\sin(A-B)$.

$\cos A = \frac{7}{25}$ (A acute), $\sin B = \frac{5}{13}$ (B obtuse), so $\tan A = \frac{24}{7}$, $\tan B = -\frac{5}{12}$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{24}{7} + (-\frac{5}{12})}{1 - \frac{24}{7} \times (-\frac{5}{12})} = \frac{253}{204}$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B = \frac{24}{25} \times (-\frac{12}{13}) - \frac{7}{25} \times \frac{5}{13} = -\frac{323}{325}$$

Answer: $\tan(A+B) = \frac{253}{204}$, $\sin(A-B) = -\frac{323}{325}$

5 Binomial expansion: rational powers (All boards)

Expand $(1 + 2x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^2.
State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \ldots$ with $n = \frac{1}{3}$, $y = 2x$

Coefficients of 1, x, x^2: 1, $\frac{2}{3}$, $-\frac{4}{9}$

Valid when $|2x| < 1$, i.e. $|x| < \frac{1}{2}$

Answer: $1 + \frac{2}{3}x - \frac{4}{9}x^2$; $|x| < \frac{1}{2}$

Day 140 **A Level Maths: A Tier 10-a-day: Model Solutions**

6 Pulled at an angle on rough ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 5 kg box is pulled along rough horizontal ground, $\mu = \frac{1}{3}$, by a rope at angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. The tension is 35 N. Find (a) the normal reaction, (b) the acceleration of the box. Take g = 9.8.

$\sin\theta = 0.8$, $\cos\theta = 0.6$
(a) Vertical: $R + 35\sin\theta = 5g$, so $R = 21$
(b) Horizontal: $35\cos\theta - \mu R = 5a$

$21 - \frac{1}{3}\times 21 = 5a$

$a = 2.8$

Answer: (a) 21 N (b) 2.8 m s^{-2}

7 Estimating mean and standard deviation (All boards)

The table summarises the mass, m grams, of eggs.
Estimate the mean and standard deviation (3 s.f.).

Midpoints: 50, 70, 90, 110, 130 (n = 57)

$\Sigma fx = 5110$, so mean $= \frac{5110}{57} = 89.6$

$\Sigma fx^2 = 524900$

$\text{sd} = \sqrt{\frac{524900}{57} - (89.6)^2} = 34.2$

Answer: mean 89.6, sd 34.2

8 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (16 kg, 1.5 m) is smoothly hinged at A to a vertical wall, with a 4 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, $\tan\theta = \frac{4}{3}$. Find (a) the tension, (b) the magnitude of the hinge force. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about A: $T\sin\theta \times 1.5 = 16g \times 0.75 + 4g \times 1.5$
$T\sin\theta = (8 + 4)g = 117.6$, so (a) $T = 147$
Horizontal component of hinge force: $X = T\cos\theta = 88.2$
Vertical: $Y + T\sin\theta = (16 + 4)g$, so $Y = 16g \div 2 = 78.4$
(b) Force $= \sqrt{(X^2 + Y^2)} = \sqrt{(88.2^2 + 78.4^2)}$

Answer: (a) 147 N (b) 118 N

9 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix}-1\\2\\-5\end{pmatrix} + t\begin{pmatrix}0\\-2\\3\end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(1, 0, −2) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathbf{PF} = \begin{pmatrix}-2\\-2t+2\\3t-3\end{pmatrix}$

$\mathbf{PF}\cdot\mathbf{d} = 0$ gives $13t - 13 = 0$, so $t = 1$

F = (−1, 0, −2); $\mathbf{PF} = \begin{pmatrix}-2\\0\\0\end{pmatrix}$, so $|\mathbf{PF}| = \sqrt{4} = 2$

Answer: F(−1, 0, −2), distance 2

10 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(38, p)$. H_0: $p = 0.4$ is tested against H_1: $p > 0.4$ using a 5% significance level.

(a) Find the critical region. (b) State the probability of a Type I error.

(c) Find the probability of a Type II error when $p = 0.6$ (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \geq 21$

(b) Type I error = P(reject H_0 | $p = 0.4$) = 0.0409

(c) Type II error = P(not in critical region | $p = 0.6$)

$P(X \leq 20)$ when $p = 0.6$: 0.222

Answer: (a) $X \geq 21$ (b) 0.0409 (c) 0.222