Day 139 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Inverse normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the journey time (minutes), is N(29, 6^2). The highest 30% of values exceed k.
Find k to 3 significant figures.

P(X > k) = 0.3, so P(X < k) = 0.7
$z = \Phi^{-1}(0.7) = 0.524$
$k = 29 + 0.524 \times 6 = 32.1$
Answer: 32.1

2 Quadratic from its graph (All boards)

The diagram shows the curve with equation y = f(x), where f is a quadratic.
The turning point is marked, and the curve crosses the y-axis as shown.
Find f(x), giving it in the form $ax^2 + bx + c$.

Turning point $(-2, -6)$: $f(x) = a(x + 2)^2 - 6$
$f(0) = -14$: $a(-2)^2 - 6 = -14$
a = −2, then expand
Answer: $f(x) = -2x^2 - 8x - 14$

3 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

The 800 items in a list are numbered 1 to 800.
A systematic sample of 40 items is taken, starting with a random item among the first few.
The first item selected is number 20.
Find the sampling interval and the number of the last item selected.

Interval = 800 ÷ 40 = 20
Items selected: 20, 40, 60, ...
Last = 20 + (40 − 1) × 20 = 800
Answer: Interval 20, last item 800

4 Transformations of a turning point (All boards)

The curve y = f(x) has a minimum point at (−3, 3).
Find the coordinates of the corresponding point on:
(a) y = 2f(x)
(b) y = −f(x)
(c) y = f(−x)

(a) y = 2f(x): (−3, 6)
(b) y = −f(x): (−3, −3)
(c) y = f(−x): (3, 3)
A stretch or translation moves the point in the stated direction only.
Answer: (a) (−3, 6); (b) (−3, −3); (c) (3, 3)

5 Beam on two supports (AQA, Edexcel, OCR, MEI)

A uniform beam AB rests on supports at C and D, with a particle attached at B. Find the reactions at C and D. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about C: $R_D \times 6 = 30\,g \times 2 + 8\,g \times 7$
$R_D = 189$
Resolving vertically: $R_C + R_D = 38\,g = 372.4$
$R_C = 372.4 - 189$
Answer: $R_C = 183$ N, $R_D = 189$ N

6 Area and perimeter of a segment (exact) (All boards)

The circle has centre O and radius 12 cm. Angle AOB = $\frac{\pi}{2}$.

Find the exact area and the exact perimeter of the shaded segment.

Sector area = $\frac{1}{2}r^2\theta = 36\pi$; triangle OAB = $\frac{1}{2}r^2\sin\theta = 72$

Segment area = sector − triangle = $36\pi - 72$

Chord AB = $2r\sin\frac{\theta}{2} = 12\sqrt{2}$; arc = $r\theta = 6\pi$

Perimeter = arc + chord = $12\sqrt{2} + 6\pi$

Answer: area = $36\pi - 72$ cm^2, perimeter = $12\sqrt{2} + 6\pi$ cm

7 Circle through two points with centre on a line (AQA, Edexcel, OCR, MEI, Eduqas)

A circle passes through A(−9, −9) and B(16, −4).
Its centre lies on the line $y = 3x - 13$.
Find the equation of the circle.

Centre $(h, 3h - 13)$ is equidistant from A and B.

$(h + 9)^2 + (3h - 4)^2 = (h - 16)^2 + (3h - 9)^2$

Solving gives $h = 3$, so centre $(3, -4)$

$r^2 = (-12)^2 + (-5)^2 = 169$

Answer: $(x - 3)^2 + (y + 4)^2 = 169$

8 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^2 - 3$. Show that the Newton-Raphson formula gives $x_{n+1} = \frac{1}{2}(x_n + \frac{3}{x_n})$,

and use it with $x_0 = 3$ to find x_2 as an approximation to $\sqrt{3}$, to 4 d.p.

$f'(x) = 2x$, so $x_{n+1} = x_n - \frac{x_n^2 - 3}{2x_n} = \frac{1}{2}(x_n + \frac{3}{x_n})$

$x_1 = \frac{1}{2}(3 + \frac{3}{3}) = 2$

$x_2 = 1.7500$

Answer: $x_2 = 1.7500$

9 Connected particles (rough table, string slack) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (3 kg) lies on a rough table, $\mu = 0.1$, joined by a light string over a smooth pulley to particle B (7 kg), 0.8 m above the floor. Released from rest; B lands without rebounding. Find (a) the acceleration of A before B lands, (b) the further distance A moves. Take $g = 9.8$.

B: $7g - T = 7a$; A: $T - 0.1\times3g = 3a$

Adding: $(7 - 0.1\times3)g = 10a$, so $a = 6.566$

Speed of B and A as B lands: $v^2 = 2\times6.566\times0.8 = 10.5056$

After B lands, A decelerates at $\mu g = 0.1\times9.8 = 0.98$

$0 = v^2 - 2(\mu g)s$, $s = 10.5056 \div 1.96$

Answer: (a) 6.566 m s^{-2} (b) 5.36 m

10 Greatest and least values using R-form (All boards)

$g(\theta) = \frac{1}{21 + 15\sin\theta + 8\cos\theta}$

(a) Write $15\sin\theta + 8\cos\theta$ in the form $R\sin(\theta + \alpha)$, with α acute.

(b) Find the greatest and least values of $g(\theta)$ and the smallest positive θ at which the greatest value occurs (1 d.p.).

$R = \sqrt{15^2 + 8^2} = 17$, $\alpha = \tan^{-1}\frac{8}{15} = 28.1°$

Denominator $= 21 + 17\sin(\theta + 28.1°)$, ranging from 4 to 38

Greatest $g = \frac{1}{4}$ (denominator least), least $g = \frac{1}{38}$

Least denominator when $\sin(\theta + 28.1°) = -1$: $\theta + 28.1° = 270°$

$\theta = 241.9°$

Answer: $R = 17$, $\alpha = 28.1°$; greatest $\frac{1}{4}$, least $\frac{1}{38}$; $\theta = 241.9°$