Day 138	A Level Maths: A Tier 10-a-day
1 A(10, −10) and B(−2, 6) are the ends of a diameter of a circle. (a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$. (b) Show that C(4, 8) lies on the circle.	
2 Find the exact value of $\int_0^{\frac{\pi}{2}} (4\sin 2x + 6\cos x)\, dx$.	
3 Forces $F_1 = (-6\mathbf{i} - 2\mathbf{j})$ N, $F_2 = (15\mathbf{i} - 8\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 5 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration.	
4 X is normally distributed with mean 68 and standard deviation σ. Given that P(X < 64) = 0.3085, find σ to 3 significant figures.	
5 A non-uniform rod AB (length 4 m, mass 4 kg) has its centre of mass at G, AG = 1.25 m. It hangs horizontally from vertical strings at A and B. A particle of mass m kg is fixed at P, AP = 3 m, making the tensions equal. Find (a) m, (b) the tension. Take g = 9.8.	

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6 $f(x) = x^2 - 3\cos x - 7$. Show that the equation $f(x) = 0$ has a root α in the interval [2, 3] (x in radians).	
7 10% of people have a fault. The inspection test is positive for 80% of those who have it and for 5% of those who do not. (a) Find the probability that a person tests positive. (b) Given a positive test, find the probability that the person has the fault (3 s.f.).	
8 Prove by contradiction that there are no integers m and n such that $6m + 15n = 98$.	
9 $f(x) = x^2 - 4x + 9$, $x \geq 2$. Find $f^{-1}(x)$ and state its domain.	
10 Water drains from a tank so that the depth h metres falls at a rate proportional to $\sqrt{h}$. Initially $h = 36$, and after 6 hours $h = 16$. Form and solve a differential equation to find how long, in total, the tank takes to empty.	