Day 137 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Conservation of momentum (CIE)

Particles A (3 kg) and B (5 kg) move in the same direction on a smooth horizontal line at 7 m s^{-1} and 7 m s^{-1}. They collide and coalesce. Find (a) the speed of the combined particle, (b) the kinetic energy lost in the collision.

Conservation of momentum (A's direction positive):
$3\times7 + 5\times7 = 8\,v$, so $v = 7$
KE before = $\frac{1}{2}\times3\times7^2 + \frac{1}{2}\times5\times7^2 = 196$
KE after = $\frac{1}{2}\times8\times7^2 = 196$
Loss = $196 - 196$
Answer: (a) 7 m s^{-1} (b) 0 J

2 Index equations solved with logarithms (All boards)

Solve $2^{x+3} = 3^{3x}$.
Give your answer to 3 significant figures.

Take logs: $(x + 3)\ln 2 = 3x \ln 3$
Collect x terms: $x(\ln 2 - 3\ln 3) = -3\ln 2$
$x = \dfrac{-3\ln 2}{\ln 2 - 3\ln 3}$
$x = 0.799$
Answer: x = 0.799

3 Polynomial division (All boards)

Find the quotient and the remainder when $x^4 + 5x^3 + 8x^2 + 24x + 17$ is divided by $(x^2 + 4)$.

Quotient (matching leading terms in turn): $x^2 + 5x + 4$
$(x^2 + 4)(x^2 + 5x + 4) = x^4 + 5x^3 + 8x^2 + 20x + 16$
Remainder = $x^4 + 5x^3 + 8x^2 + 24x + 17 - (x^4 + 5x^3 + 8x^2 + 20x + 16) = 4x + 1$
Answer: quotient $x^2 + 5x + 4$, remainder $4x + 1$

4 Differentiating a^x (All boards)

$y = 2 \times 3^{2x}$.
The gradient of the curve at P is 2916 ln 3. Find the x-coordinate of P.

$\dfrac{dy}{dx} = 4\ln 3 \times 3^{2x}$
$4\ln 3 \times 3^{2x} = 2916 \ln 3$, so $3^{2x} = 729$
$2x = 6$, so $x = 3$
Answer: x = 3

5 Cubic inequalities (All boards)

Given that $(x - 3)$ is a factor of $f(x) = x^3 - 5x^2 - 2x + 24$, solve $f(x) < 0$.

Factorise: $f(x) = (x + 2)(x - 3)(x - 4)$
Roots at $x = -2, 3, 4$
Sketch the cubic (positive x^3 coefficient: negative, positive, negative, positive)
Read off where $f(x) < 0$.
Answer: $x < -2$ or $3 < x < 4$

6 Poisson over different intervals (CIE)

Faults are reported at random at an average of 3 per hour.
Find the probability that in a 90-minute period there are (a) zero events (b) at least 2.
(c) Find the probability that at least one occurs in each of two separate 90-minute periods.

$\lambda = 3 \times \dfrac{90}{60} = 4.5$, $X \sim Po(4.5)$
(a) $P(X = 0) = e^{-4.5} = 0.0111$
(b) $P(X \geq 2) = 1 - P(X \leq 1) = 0.939$
(c) $(1 - 0.0111)^2 = 0.978$
Answer: (a) 0.0111 (b) 0.939 (c) 0.978

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7 Position vector and velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has position vector $\mathbf{r} = (t^3 + 2t)\mathbf{i} + (t^2 - 2t)\mathbf{j}$ metres at time t s. Find (a) the speed of P when t = 2, (b) the value of t when P is moving parallel to $\mathbf{i}$. Give answers to 3 s.f. if not exact.

$\mathbf{v} = d\mathbf{r}/dt = (3t^2 + 2)\mathbf{i} + (2t - 2)\mathbf{j}$

(a) t = 2: $\mathbf{v} = 14\mathbf{i} + 2\mathbf{j}$

Speed = $\sqrt{(14^2 + 2^2)} = \sqrt{200}$

(b) Parallel to $\mathbf{i}$ means the $\mathbf{j}$ component is zero: $2t - 2 = 0$, so t = 1

Answer: (a) 14.1 m s^{-1} (b) t = 1

8 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass of an egg is N(43, 5^2). An egg is classed as 'special' if its mass is less than 49.
8 eggs are chosen at random. Find the probability that exactly 3 are special (3 s.f.).

P(special) = P(X < 49) = P(Z < 1.20) = 0.885

Y ~ B(8, 0.885)

$P(Y = 3) = C(8, 3)(0.885)^3(0.115)^5 = 0.000783$

Answer: 0.000783

9 Tangent conditions (All boards)

The line $y = kx - 3$ is a tangent to the curve $y = x^2 + 4x - 2$.
(a) Find the possible values of k.
(b) For the larger value of k, find the coordinates of the point of contact.

Equate: $x^2 + 4x - 2 = kx - 3$

$x^2 + (4 - k)x + 1 = 0$

Tangent: discriminant = 0: $(4 - k)^2 = 4$

$4 - k = \pm 2$ so k = 6 or 2

k = 6: $x = \frac{k - 4}{2} = 1$, y = 3

Answer: (a) k = 2 or k = 6 (b) (1, 3)

10 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_1^4 \left(\frac{2}{x} + \frac{7}{x^2}\right) dx$.

Integrate: $[2 \ln x - \frac{7}{x}]$

Substitute the limits and subtract: $\frac{16 \ln 2 + 21}{4}$

Answer: $\frac{16 \ln 2 + 21}{4}$