Day 136 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A company employs 192 people: 48 Sales, 56 Admin and 88 Production.
A stratified sample of size 24 is taken.
Find the number of people sampled from each group.

Sampling fraction = $\frac{24}{192} = \frac{1}{8}$

Sales: $48 \times \frac{1}{8} = 6$

Admin: $56 \times \frac{1}{8} = 7$

Production: $88 \times \frac{1}{8} = 11$

Answer: Sales 6, Admin 7, Production 11

2 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 34.3 m s^{-1} from a point h m above horizontal ground, and hits the ground 11 s later.
Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: $s = -h$, $u = 34.3$, $a = -9.8$, $t = 11$
$-h = 34.3\times11 - 4.9\times11^2$, so $h = 215.6$
(b) At the highest point $v = 0$: $0 = 34.3^2 - 2\times9.8\times s$
$s = 60.025$ m above the point of projection
Greatest height above the ground = 215.6 + 60.025
Answer: (a) 215.6 m (b) 275.625 m

3 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean windspeed (knots) in the same month are:
Camborne: mean 17, standard deviation 2.6;
Leeming: mean 7, standard deviation 3.7.
Make two comparisons.

Means: Camborne has the higher mean (17 against 7), so on average the mean windspeed is greater at Camborne
Standard deviations: Leeming is larger at Leeming (3.7 against 2.6)
so the readings at Leeming are more variable and those at Camborne are more consistent
Answer: Camborne higher on average; Leeming more variable

4 Product rule: stationary points (All boards)

$y = (3x + 6)e^{-x}$.
Find the exact coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 3e^{-x} - (3x + 6)e^{-x} = (-3x - 3)e^{-x}$

$e^{-x} > 0$, so $\frac{dy}{dx} = 0$ gives $-3x - 3 = 0$ and $x = -1$

$y = 3e$; $\frac{d^2y}{dx^2}$ is negative there, so it is a maximum

Answer: $(-1, 3e)$, maximum

5 Binomial expansion: rational powers (All boards)

Expand $(1 - \frac{x}{2})^{-1}$ in ascending powers of x up to and including the term in x^3.

State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \ldots$ with $n = -1$, $y = -\frac{x}{2}$

Coefficients of 1, x, x^2, x^3: 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$

Valid when $|\frac{x}{2}| < 1$, i.e. $|x| < 2$

Answer: $1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3$; $|x| < 2$

6 Optimisation problems (All boards)

A closed cylinder has radius r cm and volume 2π cm^3.

Show that its surface area is $A = 2\pi r^2 + \frac{4\pi}{r}$, and find the minimum value of A in terms of π.

$h = \frac{2}{r^2}$, so $A = 2\pi r^2 + 2\pi rh = 2\pi r^2 + \frac{4\pi}{r}$

$\frac{dA}{dr} = 4\pi r - \frac{4\pi}{r^2} = 0$, so $r^3 = 1$ and $r = 1$

$\frac{d^2A}{dr^2} = 4\pi + \frac{8\pi}{r^3} > 0$, so a minimum

$A = 2\pi \times 1 + \frac{4\pi}{1} = 6\pi$

Answer: $A_{min} = 6\pi$ cm^2 ($r = 1$)

7 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is -9 and the coefficient of x^2 is 36. Find n and k.

$nk = -9$ and $\frac{n(n-1)}{2}k^2 = 36$

$(n - 1)k = \frac{2 \times 36}{-9} = -8$ (dividing the second equation by the first)

$n\,k - (n - 1)\,k = k = -9 - (-8) = -1$

$n = \frac{-9}{-1} = 9$

Answer: $n = 9$, $k = -1$

8 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $2x^2 + 2xy + 2y^2 = 54$.

Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{4x + 2y}{2x + 4y}$

Horizontal tangent: $4x + 2y = 0$, so $y = -2x$

Substitute: $6x^2 = 54$, so $x^2 = 9$ and $x = \pm 3$

Points: $(3, -6)$ and $(-3, 6)$

Answer: $(3, -6)$ and $(-3, 6)$

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9 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle α above the horizontal, $\tan\alpha = \frac{3}{4}$, and passes through A, 16 m horizontally from O and 7.1 m above the ground. Find the speed of projection. Take g = 9.8.

Let the speed be U. Horizontal: $16 = U\cos\alpha \times t$, so $t = 16 \div (0.8\,U)$
Vertical: $7.1 = U\sin\alpha \times t - 4.9t^2$
Eliminate t: $y = x\tan\alpha - 4.9x^2 \div (U^2\cos^2\alpha)$
$7.1 = 12 - 4.9\times16^2 \div (0.64\,U^2)$
$U^2 = 1960 \div 4.9 = 400$, $U = 20$

Answer: 20 m s^{-1}

10 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{5} - 2) = -2 - 4\sqrt{5}$, writing x in the form $p + q\sqrt{5}$.

$$x = \frac{-2 - 4\sqrt{5}}{\sqrt{5} - 2}$$

Multiply top and bottom by $\sqrt{5} + 2$.
Numerator: $(-2 - 4\sqrt{5})(\sqrt{5} + 2) = -24 - 10\sqrt{5}$
Denominator: $5 - 4 = 1$

Answer: $x = -24 - 10\sqrt{5}$