Day 134 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\sin^2 x + \cos x - 2 = 0$
for $-180° \leq x \leq 180°$.

Use $\sin^2 x \equiv 1 - \cos^2 x$: $2\cos^2 x - \cos x = 0$

Solving the quadratic: $\cos x = 0$ or $\cos x = \frac{1}{2}$

$\cos x = 0$: $x = -90°, 90°$

$\cos x = \frac{1}{2}$: $x = -60°, 60°$

Answer: $x = -90°, -60°, 60°, 90°$

2 Transformations from a graph (All boards)

The diagram shows the curve $y = f(x)$, with turning points A and B.
Find the coordinates of the turning points of
(a) $y = -f(x)$ (b) $y = f(x + 2)$

(a) $y = -f(x)$: A → $(0, -4)$, B → $(2, 4)$
(b) $y = f(x + 2)$: A → $(-2, 4)$, B → $(0, -4)$
Answer: (a) $(0, -4)$, $(2, 4)$; (b) $(-2, 4)$, $(0, -4)$

3 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the score (marks), is normally distributed with mean 54 and standard deviation 12.
Find (a) $P(X > 28.8)$ (b) $P(28.8 < X < 37.2)$, each to 3 s.f.

(a) $z = \frac{28.8 - 54}{12} = -2.10$, $P(Z > -2.10) = 0.982$

(b) $z_1 = -2.10$, $z_2 = \frac{37.2 - 54}{12} = -1.40$
$P(28.8 < X < 37.2) = \Phi(-1.40) - \Phi(-2.10) = 0.0629$
Answer: (a) 0.982 (b) 0.0629

4 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
$3\sin 2\theta + \cos\theta = 1.5$
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin 2\theta \approx 2\theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$

$3 \times 2\theta + 1(1 - \frac{1}{2}\theta^2) = 1.5$

Multiply by 2 and rearrange: $\theta^2 - 12\theta + 1 = 0$

$\theta = \frac{12 \pm \sqrt{140}}{2}$, so $\theta = 0.0839$

Answer: $a = 1$, $b = -12$, $c = 1$; $\theta \approx 0.0839$

5 Equation of a reciprocal curve from its graph (All boards)

The diagram shows the curve $y = \frac{k}{x - a} + b$ with asymptotes as shown. Find the values of k, a and b, and the coordinates where the curve crosses the axes.

Asymptotes $x = 2$, $y = -3$ so $a = 2$, $b = -3$

Through $(3, 0)$: $0 = \frac{k}{1} - 3$

$k = (0 + 3)(1) = 3$

$x = 0$: $y = -\frac{9}{2}$; $y = 0$: $x = 3$

Answer: $k = 3$, $a = 2$, $b = -3$; $(0, -\frac{9}{2})$ and $(3, 0)$

6 Velocity, rest and total distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on the x-axis with velocity $v = (3t^2 - 24t + 45)$ m s^{-1} at time t s, $t \geq 0$, and starts at O. Find (a) the times when P is at instantaneous rest, (b) the total distance P travels in the first 8 s.

(a) $v = 3(t - 3)(t - 5) = 0$, so $t = 3$ or $t = 5$

(b) Displacement $x = \int v\,dt = t^3 - 12t^2 + 45t$

$x(3) = 54$, $x(5) = 50$, $x(8) = 104$

Direction changes at $t = 3$ and $t = 5$, so distance $= |54| + |50 - 54| + |104 - 50|$

Answer: (a) $t = 3$, $t = 5$ (b) 112 m

7 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{2} - 2) = 4 + 3\sqrt{2}$, writing x in the form $p + q\sqrt{2}$.

$$x = \frac{4 + 3\sqrt{2}}{\sqrt{2} - 2}$$

Multiply top and bottom by $\sqrt{2} + 2$.

Numerator: $(4 + 3\sqrt{2})(\sqrt{2} + 2) = 14 + 10\sqrt{2}$

Denominator: $2 - 4 = -2$

Answer: $x = -7 - 5\sqrt{2}$

8 Smooth incline: passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected up a line of greatest slope of a smooth plane inclined at α. It passes A, 29.4 m from the start, after 4 s and again after 6 s. Find (a) the speed of projection, (b) α to 1 d.p. Take $g = 9.8$.

Deceleration $a = g \sin\alpha$ (unknown). $s = ut - \frac{1}{2}at^2$ at both times:

$u \times 4 - \frac{1}{2}a \times 4^2 = u \times 6 - \frac{1}{2}a \times 6^2 = 29.4$

Subtracting: $u(6 - 4) = \frac{1}{2}a(36 - 16)$, so $u = \frac{1}{2}a(10)$

Substituting: $\frac{1}{2}a(10)(4) - \frac{1}{2}a \times 4^2 = 29.4$, so $\frac{1}{2}a \times 24 = 29.4$, $a = 2.45$

(a) $u = 1.225 \times 10 = 12.25$

(b) $9.8 \sin\alpha = 2.45$, $\sin\alpha = 0.25$, $\alpha = 14.5°$

Answer: (a) 12.25 m s^{-1} (b) 14.5°

9 Equations with cosec x and cot x (All boards)

Solve $3\text{cosec}^2 x - 6\cot x - 27 = 0$

for $-180° < x < 180°$, $x \neq 0$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $3\cot^2 x - 6\cot x - 24 = 0$

$\cot x = -2$ or $\cot x = 4$

$\cot x = -2$ means $\tan x = -\frac{1}{2}$: $x = -26.6°, 153.4°$

$\cot x = 4$ means $\tan x = \frac{1}{4}$: $x = -166.0°, 14.0°$

Answer: $x = -166.0°, -26.6°, 14.0°, 153.4°$

10 Geometric distribution (CIE)

On each independent trial, the probability that a biased die shows a six is $\frac{1}{4}$.

X is the number of trials up to and including the first success.

(a) Find P(X = 3) and E(X).

(b) Find the least m with $P(X \leq m) > 0.99$.

(a) $P(X = 3) = (\frac{3}{4})^2 \times \frac{1}{4} = \frac{9}{64}$; $E(X) = \frac{1}{p} = 4$

(b) $P(X \leq m) = 1 - (\frac{3}{4})^m > 0.99$

$(\frac{3}{4})^m < 0.01$, so $m > 16.0$

Least integer $m = 17$

Answer: (a) $\frac{9}{64}$, E(X) = 4 (b) m = 17