Day 133 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Polynomial division (All boards)

Find the quotient and the remainder when $4x^3 - 18x^2 + 16x + 15$ is divided by $(2x - 5)$.

Quotient (matching leading terms in turn): $2x^2 - 4x - 2$

$(2x - 5)(2x^2 - 4x - 2) = 4x^3 - 18x^2 + 16x + 10$

Remainder $= 4x^3 - 18x^2 + 16x + 15 - (4x^3 - 18x^2 + 16x + 10) = 5$

Answer: quotient $2x^2 - 4x - 2$, remainder 5

2 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 5 kg is pushed at constant speed along rough horizontal ground, $\mu = 0.5$, by a force of P N at angle θ below the horizontal, $\tan\theta = \frac{5}{12}$. Find P and the normal reaction. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

Constant speed, so the resultant force is zero. $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

Vertical: $R = 5g + P\sin\theta$

Horizontal: $P\cos\theta = \mu R = 0.5(5g + P\sin\theta)$

$P(\frac{12}{13} - \frac{5}{26}) = 0.5 \times 5g$

$P = 33.5$, $R = 61.9$

Answer: P = 33.5 N, R = 61.9 N

3 R-form and solving a sin x + b cos x = c (All boards)

(a) Express $6\cos\theta - 8\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give α to 1 d.p.

(b) Solve $6\cos\theta - 8\sin\theta = 5$ for $0° \leq \theta < 360°$, giving answers to 1 d.p.

$R^2 = 6^2 + 8^2 = 100$, so $R = 10$; $\tan\alpha = \frac{8}{6}$ gives $\alpha = 53.1°$

$10\cos(\theta + 53.1°) = 5$, so $\cos(\theta + 53.1°) = \frac{1}{2}$

Principal value 60.0°, then use the symmetry of the cosine graph

$\theta = 6.9°, 246.9°$

Answer: (a) R = 10, α = 53.1° (b) θ = 6.9°, 246.9°

4 Inverse normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the journey time (minutes), is $N(39, 6^2)$. The highest 15% of values exceed k.

Find k to 3 significant figures.

$P(X > k) = 0.15$, so $P(X < k) = 0.85$

$z = \Phi^{-1}(0.85) = 1.04$

$k = 39 + 1.04 \times 6 = 45.2$

Answer: 45.2

5 Area between curves and the x-axis (with sketch) (All boards)

The diagram shows the curve $y = x(x - 3)(x - 6)$.

Find the total area of the two shaded regions.

$y = x^3 - 9x^2 + 18x$; $\int y\,dx = \frac{1}{4}x^4 - 3x^3 + 9x^2$

From 0 to 3: $\frac{81}{4}$ (above the axis)

From 3 to 6: $-\frac{81}{4}$ (below the axis), so the area is $\frac{81}{4}$

Total $= \frac{81}{4} + \frac{81}{4} = \frac{81}{2}$

Answer: $\frac{81}{2}$

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6 Binomial expansion: product with a bracket (All boards)

Find the coefficient of x^4 in the expansion of
$(3 + 3x)(1 + x)^5$.

In $(1 + x)^5$: coefficient of x^3 is 10, of x^4 is 5
Coefficient = $3 \times 5 + 3 \times 10$
Answer: 45

7 Regression with coded variables (AQA, Edexcel, OCR, MEI, Eduqas)

A regression line of v on w is $v = 1003.5 + 5w$, where $w = \frac{t - 2000}{10}$.
(a) Find the regression line of v on t in the form $v = p + qt$.
(b) Estimate v when $t = 2060$.

$v = 1003.5 + 5 \times \frac{t - 2000}{10}$
$= 1003.5 + 0.5t - 1000 = 3.5 + 0.5t$
(b) $v = 3.5 + 0.5 \times 2060 = 1033.5$
Answer: (a) $v = 3.5 + 0.5t$ (b) 1033.5

8 Separable differential equations (All boards)

Solve the differential equation
$x\frac{dy}{dx} = 4y$,
given that $y = 3$ when $x = 1$ ($x > 0$, $y > 0$). Give y in terms of x.

Separate: $\int \frac{1}{y}\, dy = \int \frac{4}{x}\, dx$, so $\ln y = 4 \ln x + C$
$x = 1$, $y = 3$ gives $C = \ln 3$
$\ln y = \ln(3x^4)$, so $y = 3x^4$
Answer: $y = 3x^4$

9 Modulus inequalities (All boards)

Solve $|4x - 6| > |-2x - 3|$.

Square both sides: $(4x - 6)^2 > (-2x - 3)^2$
$((4x - 6) - (-2x - 3))((4x - 6) + (-2x - 3)) > 0$
$(6x - 3)(2x - 9) > 0$
Critical values $x = \frac{1}{2}$ and $x = \frac{9}{2}$; sketch the (positive or negative) quadratic.
Answer: $x < \frac{1}{2}$ or $x > \frac{9}{2}$

10 Closest approach of two particles (Edexcel, OCR, MEI)

At t = 0, P and Q have position vectors $(2\mathbf{i} + 11\mathbf{j})$ m and $(\mathbf{i} + 3\mathbf{j})$ m and velocities $(-4\mathbf{i} - 4\mathbf{j})$ m s^{-1} and $(-2\mathbf{i} - 3\mathbf{j})$ m s^{-1}.
Find (a) the time when they are closest, (b) the least distance between them. Give answers to 3 s.f. if not exact.

$PQ = r_P - r_Q = (\mathbf{i} + 8\mathbf{j}) + t(-2\mathbf{i} - \mathbf{j})$
$d^2 = (-2t + 1)^2 + (-t + 8)^2 = 5t^2 - 20t + 65$
(a) $d(d^2)/dt = 0$: $10t + -20 = 0$, so $t = 2$
(b) At $t = 2$: $PQ = -3\mathbf{i} + 6\mathbf{j}$, so $d = \sqrt{45}$
Answer: (a) t = 2 s (b) 6.71 m