Day 132 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 2 m s^{-1} and accelerates uniformly. She travels 160 m in the next 20 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ.

(a) $s = ut + \frac{1}{2}at^2$: $160 = 2\times20 + \frac{1}{2}\times a\times20^2$, so $a = 0.6$
(b) $v = u + at = 2 + 0.6\times20 = 14$
(c) For the midpoint, $s = 80$: $v^2 = u^2 + 2as = 2^2 + 2\times0.6\times80$
$v^2 = 100$
Answer: (a) 0.6 m s^{-2} (b) 14 m s^{-1} (c) 10 m s^{-1}

2 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^2 in the expansion of $(-2 + 2x)^6$.

Term in x^2: ${}^6C_2 \times (-2)^4 \times (2x)^2$
$= 15 \times 16 \times 4$
Answer: 960

3 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the journey time (minutes), is normally distributed with mean 35 and standard deviation 6.
Find (a) P(X > 26.6) (b) P(26.6 < X < 30.2), each to 3 s.f.

(a) $z = \frac{26.6 - 35}{6} = -1.40$, $P(Z > -1.40) = 0.919$

(b) $z_1 = -1.40$, $z_2 = \frac{30.2 - 35}{6} = -0.800$
$P(26.6 < X < 30.2) = \Phi(-0.800) - \Phi(-1.40) = 0.131$
Answer: (a) 0.919 (b) 0.131

4 Change of sign to locate a root (All boards)

$f(x) = \ln x + x - 8$.
Show that the equation $f(x) = 0$ has a root α in the interval [6, 7].

f(x) is continuous on the interval
$f(6) = -0.208 < 0$
$f(7) = 0.946 > 0$
A change of sign, so a root lies in [6, 7]
Answer: Sign change: f(6) = −0.208, f(7) = 0.946

5 Arithmetic series: S_n formula (All boards)

The sum of the first n terms of an arithmetic series is $S_n = 3n^2 + 5n$.
(a) Find the first term and the common difference.
(b) Find u_{29}.

$u_1 = S_1 = 8$
$S_2 = 22$, so $u_2 = 14$
$d = u_2 - u_1 = 6$
$u_{29} = 8 + 28 \times 6$
Answer: (a) a = 8, d = 6 (b) 176

6 Ships on collision courses (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

At 12:00 ship P has position $(-14\mathbf{i} - 10\mathbf{j})$ km and velocity $(4\mathbf{i} + 5\mathbf{j})$ km h^{-1}; ship Q has position $(-12\mathbf{i} + 8\mathbf{j})$ km and velocity $(3\mathbf{i} - 4\mathbf{j})$ km h^{-1}. Show that the ships collide and find the time and position vector of the collision.

At time t hours: $\mathbf{r}_P = (-14\mathbf{i} - 10\mathbf{j}) + t(4\mathbf{i} + 5\mathbf{j})$, $\mathbf{r}_Q = (-12\mathbf{i} + 8\mathbf{j}) + t(3\mathbf{i} - 4\mathbf{j})$
Equal when: **i**: $-14 + 4t = -12 + 3t$, so $t = 2$
j: $-10 + 2\times5 = 0$
Check with Q: **i**: $-12 + 2\times3 = -6$, **j**: $8 + 2\times(-4) = 0$
Both components agree at t = 2, so they collide at $(-6\mathbf{i})$ km
Answer: t = 2 hours (at 14:00); r = (−6i) km

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7 Equation of a reciprocal curve from its graph (All boards)

The diagram shows the curve $y = \frac{k}{x - a} + b$ with asymptotes as shown. Find the values of k, a and b, and the coordinates where the curve crosses the axes.

Asymptotes $x = 3$, $y = 2$ so $a = 3$, $b = 2$

Through (4, 3): $3 = \frac{k}{1} + 2$

$k = (3 - 2)(1) = 1$

$x = 0$: $y = \frac{5}{3}$; $y = 0$: $x = \frac{5}{2}$

Answer: $k = 1$, $a = 3$, $b = 2$; $(0, \frac{5}{3})$ and $(\frac{5}{2}, 0)$

8 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $3x^2 - 2xy + 2y^2 = 60$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{6x - 2y}{-2x + 4y}$

Horizontal tangent: $6x - 2y = 0$, so $y = 3x$

Substitute: $15x^2 = 60$, so $x^2 = 4$ and $x = \pm 2$

Points: (2, 6) and $(-2, -6)$

Answer: (2, 6) and $(-2, -6)$

9 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against x for $y = A\,b^x$.
Find the exact values of A and b.

$\ln y = \ln A + x \ln b$, a straight line with gradient $\ln b$ and intercept $\ln A$

Intercept $\ln A = 3$, so $A = e^3$

Gradient $\ln b = \frac{1 - 3}{4 - 0} = -\frac{1}{2}$, so $b = e^{-\frac{1}{2}}$

Answer: $A = e^3$, $b = e^{-\frac{1}{2}}$

10 Regression line from summary data (AQA, Edexcel, OCR, MEI, Eduqas)

n = 9 pairs of (x, y): $\Sigma x = 137$, $\Sigma y = 408$, $\Sigma x^2 = 2735$, $\Sigma xy = 7156$.
(a) Find the equation of the regression line of y on x (3 s.f.).
(b) Estimate y when $x = 14$.

$S_{xx} = 2735 - \frac{137^2}{9} = 650$, $S_{xy} = 7156 - \frac{137 \times 408}{9} = 945$

$b = \frac{S_{xy}}{S_{xx}} = 1.46$

$a = \bar{y} - b\,\bar{x} = 45.3 - (1.46)(15.2) = 23.2$

(b) $y = 23.2 + (1.46)(14) = 43.6$

Answer: (a) $y = 23.2 + 1.46x$ (b) 43.6