Day 131 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Transformations of a turning point (All boards)

The curve y = f(x) has a maximum point at (−4, −5).
Find the coordinates of the corresponding point on:
(a) y = f(2x)
(b) y = f(x − 3)
(c) y = 3f(x)

(a) y = f(2x): (−2, −5)
(b) y = f(x − 3): (−1, −5)
(c) y = 3f(x): (−4, −15)
A stretch or translation moves the point in the stated direction only.
Answer: (a) (−2, −5); (b) (−1, −5); (c) (−4, −15)

2 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{12}{13}$ and $\cos B = -\frac{5}{13}$.
Find the exact values of tan(A+B) and cos(A+B).

$\cos A = \frac{5}{13}$ (A acute), $\sin B = \frac{12}{13}$ (B obtuse), so $\tan A = \frac{12}{5}$, $\tan B = -\frac{12}{5}$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{12}{5} + (-\frac{12}{5})}{1 - \frac{12}{5} \times (-\frac{12}{5})} = 0$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B = \frac{5}{13} \times (-\frac{5}{13}) - \frac{12}{13} \times \frac{12}{13} = -1$$

Answer: tan(A+B) = 0, cos(A+B) = −1

3 Coding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 25 values of x, $\Sigma(x - 20) = 100$ and $\Sigma(x - 20)^2 = 625$.
Find the mean and standard deviation of x.

Let y = x − 20: $\bar{y} = \frac{100}{25} = 4$
Mean of x = 4 + 20 = 24
Variance = $\frac{625}{25} - 4^2 = 9$
Subtracting a constant does not change spread, so sd = 3
Answer: mean 24, sd 3

4 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 - 8x + 8y + 3 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P(3, 1) lies inside or outside the circle.

Complete the square: $(x - 4)^2 + (y + 4)^2 = 29$
Centre (4, −4), radius $\sqrt{29}$
Distance^2 from centre to P = $(-1)^2 + 5^2 = 26$
Compare with $r^2 = 29$
Answer: (a) centre (4, −4), radius $\sqrt{29}$ (b) P is inside the circle

5 Horizontal projection (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 15 m s^{-1} from a point 19.6 m above horizontal ground. Find (a) the horizontal distance to the landing point, (b) the speed and direction (angle below the horizontal, 1 d.p.) on landing. Take g = 9.8. Give answers to 3 s.f. if not exact.

Vertical: $19.6 = \frac{1}{2} \times 9.8 \times t^2$, so t = 2
(a) Horizontal distance = 15 × 2
(b) Vertical speed on landing = 9.8×2 = 19.6; horizontal speed = 15
Speed = $\sqrt{(15^2 + 19.6^2)} = 24.7$
$\tan \theta = 19.6 \div 15$, $\theta = 52.6°$
Answer: (a) 30 m (b) 24.7 m s^{-1} at 52.6° below the horizontal

6 R-form and solving a sin x + b cos x = c (All boards)

(a) Express 21 sin x + 20 cos x in the form R sin(x + α), where R > 0 and 0° < α < 90°. Give α to 1 d.p.
(b) Solve 21 sin x + 20 cos x = 14 for 0° ≤ x < 360°, giving answers to 1 d.p.

$R^2 = 21^2 + 20^2 = 841$, so R = 29; $\tan \alpha = \frac{20}{21}$ gives α = 43.6°

$29 \sin(x + 43.6°) = 14$, so $\sin(x + 43.6°) = \frac{14}{29}$

Principal value 28.9°, then use the symmetry of the sine graph
x = 107.5°, 345.3°
Answer: (a) R = 29, α = 43.6° (b) x = 107.5°, 345.3°

7 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{3} - 3) = -3 - 3\sqrt{3}$, writing x in the form $p + q\sqrt{3}$.

$$x = \frac{-3 - 3\sqrt{3}}{\sqrt{3} - 3}$$

Multiply top and bottom by $\sqrt{3} + 3$.
Numerator: $(-3 - 3\sqrt{3})(\sqrt{3} + 3) = -18 - 12\sqrt{3}$
Denominator: $3 - 9 = -6$
Answer: $x = 3 + 2\sqrt{3}$

8 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 20 m s^{-1} at angle α above the horizontal, $\tan \alpha = \frac{12}{5}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity $(\frac{100}{13}, \frac{240}{13})$. At time T the velocity is $(\frac{100}{13}, \frac{240}{13} - 9.8T)$.

Perpendicular: scalar product = 0: $(\frac{100}{13})^2 + \frac{240}{13}(\frac{240}{13} - 9.8T) = 0$

$20^2 - 9.8T \times \frac{240}{13} = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 20 \div (9.8 \times \frac{12}{13}) = \frac{325}{147}$

(b) $h = \frac{240}{13} \times \frac{325}{147} - 4.9 \times \frac{325}{147}^2 = \frac{2125}{126}$
Answer: (a) T = 2.21 s (b) 16.9 m

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9 Greatest and least values using R-form (All boards)

$g(\theta) = \frac{1}{36 + 20\sin\theta + 21\cos\theta}$

(a) Write $20\sin\theta + 21\cos\theta$ in the form $R\sin(\theta + \alpha)$, with α acute.

(b) Find the greatest and least values of $g(\theta)$ and the smallest positive θ at which the greatest value occurs (1 d.p.).

$R = \sqrt{20^2 + 21^2} = 29$, $\alpha = \tan^{-1}\frac{21}{20} = 46.4°$

Denominator $= 36 + 29\sin(\theta + 46.4°)$, ranging from 7 to 65

Greatest $g = \frac{1}{7}$ (denominator least), least $g = \frac{1}{65}$

Least denominator when $\sin(\theta + 46.4°) = -1$: $\theta + 46.4° = 270°$

$\theta = 223.6°$

Answer: $R = 29$, $\alpha = 46.4°$; greatest $\frac{1}{7}$, least $\frac{1}{65}$; $\theta = 223.6°$

10 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(37, p)$. H_0: $p = 0.3$ is tested against H_1: $p < 0.3$ using a 5% significance level.

(a) Find the critical region. (b) State the probability of a Type I error.

(c) Find the probability of a Type II error when $p = 0.15$ (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \leq 6$

(b) Type I error = P(reject H_0 | $p = 0.3$) = 0.0440

(c) Type II error = P(not in critical region | $p = 0.15$)

$P(X \geq 7)$ when $p = 0.15$: 0.315

Answer: (a) $X \leq 6$ (b) 0.0440 (c) 0.315