Day 130 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arc length and sector area (All boards)

A sector of a circle of radius 13 cm has perimeter 49.4 cm.
Find (a) the angle of the sector in radians, (b) the area of the sector.

Perimeter = $2r + r\theta$: $49.4 = 26 + 13\theta$

$\theta = \frac{23.4}{13} = 1.8$

Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 13^2 \times 1.8 = 152.1$

Answer: (a) 1.8 rad (b) 152.1 cm^2

2 Equations quadratic in disguise (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 7\sqrt{x} + 10 = 0$.

Let $u = \sqrt{x}$: $u^2 - 7u + 10 = 0$
$(u - 2)(u - 5) = 0$ so $u = 2$ or 5
$\sqrt{x} = 2$ gives $x = 4$; $\sqrt{x} = 5$ gives $x = 25$
Answer: $x = 4$ or $x = 25$

3 Constant acceleration in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has constant acceleration $(4\mathbf{i} + 2\mathbf{j})$ m s^{-2}. When $t = 0$, P is at $(2\mathbf{i} + 3\mathbf{j})$ m moving with velocity $(-8\mathbf{i} + 3\mathbf{j})$ m s^{-1}.
Find (a) when P moves parallel to $\mathbf{j}$, (b) its position vector then.

$\mathbf{v} = \mathbf{u} + \mathbf{a}t = (-8\mathbf{i} + 3\mathbf{j}) + t(4\mathbf{i} + 2\mathbf{j})$
(a) Parallel to $\mathbf{j}$: the $\mathbf{i}$ component is zero: $-8 + 4t = 0$, so $t = 2$
(b) $\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$= (2\mathbf{i} + 3\mathbf{j}) + 2(-8\mathbf{i} + 3\mathbf{j}) + 2(4\mathbf{i} + 2\mathbf{j})$
$= -6\mathbf{i} + 13\mathbf{j}$
Answer: (a) $t = 2$ (b) $\mathbf{r} = (-6\mathbf{i} + 13\mathbf{j})$ m

4 Discriminant: range of k (All boards)

The equation $3x^2 + (k + 6)x + (k + 6) = 0$ has two distinct real roots.
Find the set of possible values of k.

Discriminant: $b^2 - 4ac = (k + 6)^2 - 4 \times 3(k + 6)$
$= k^2 - 36$
Roots of the quadratic in k: $k = -6$ and $k = 6$
Required: $b^2 - 4ac > 0$
Answer: $k < -6$ or $k > 6$

5 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 6 kg is pulled at constant speed along rough horizontal ground, $\mu = 0.5$, by a force of P N at angle θ above the horizontal, $\tan\theta = \frac{4}{3}$. Find P and the normal reaction. Take $g = 9.8$.

Constant speed, so the resultant force is zero. $\sin\theta = 0.8$, $\cos\theta = 0.6$
Vertical: $R = 6g - P\sin\theta$
Horizontal: $P\cos\theta = \mu R = 0.5(6g - P\sin\theta)$
$P(0.6 + 0.4) = 0.5 \times 6g$
$P = 29.4$, $R = 35.3$
Answer: $P = 29.4$ N, $R = 35.28$ N

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6 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value of $\sin(\arcsin\frac{3}{5} + \arcsin\frac{8}{17})$.

Let A = $\arcsin\frac{3}{5}$ and B = $\arcsin\frac{8}{17}$, both acute

$\cos A = \frac{4}{5}$, $\cos B = \frac{15}{17}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B = \frac{3}{5} \times \frac{15}{17} + \frac{4}{5} \times \frac{8}{17} = \frac{77}{85}$

Answer: $\frac{77}{85}$

7 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Parts are defective with probability 0.15, independently. A box of 9 parts is rejected if it has at least 3 defective. From 12 such boxes, find the probability that exactly 3 are rejected (3 s.f.).

$Y \sim B(9, 0.15)$. $P(\text{rejected}) = P(Y \geq 3) = 1 - 0.859 = 0.141$
$W \sim B(12, 0.141)$ is the number rejected
$P(W = 3) = C(12, 3)(0.141)^3(1 - 0.141)^9 = 0.157$
Answer: 0.157

8 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix} -4 \\ -1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $4 + 2\lambda = -4 - 2\mu$; $1 + 1\lambda = -1 + 0\mu$; $-3 - 2\lambda = 4 + 2\mu$
Solving two of the equations gives a unique (λ, μ)
which does not satisfy the third, so the lines do not meet
Answer: The lines do not intersect (skew)

9 Factor theorem: roots with surds (All boards)

$f(x) = x^3 - x^2 - 11x - 10$.
(a) Show that (x + 2) is a factor of f(x).
(b) Solve f(x) = 0, giving surds in simplest form.

$f(-2) = -8 - 4 + 22 - 10 = 0$, so (x + 2) is a factor.

$f(x) = (x + 2)(x^2 - 3x - 5)$

$x^2 - 3x - 5 = 0$ gives $x = \frac{3 \pm \sqrt{29}}{2}$

Answer: $x = -2$ **or** $x = \frac{3 \pm \sqrt{29}}{2}$

10 Conditional probability without replacement (All boards)

A bag has 2 red and 6 blue counters. Two are taken at random without replacement.
Given that the counters are the same colour, find the probability they are both red.

P(both red) = $\frac{2}{8} \times \frac{1}{7} = \frac{1}{28}$

P(both blue) = $\frac{6}{8} \times \frac{5}{7} = \frac{15}{28}$

P(same colour) = $\frac{4}{7}$

P(both red | same) = $\frac{\frac{1}{28}}{\frac{4}{7}}$

= $\frac{1}{16}$

Answer: $\mathbf{\frac{1}{16}}$