Day 13 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Bayes' theorem in context (All boards)

0.5% of people have a disease. The screening test is positive for 80% of those who have it and for 5% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the disease (3 s.f.).

(a) P(+) = 0.005 × 0.8 + 0.995 × 0.05
= 0.004 + 0.04975 = 0.05375

(b) P(has it | +) = $\frac{0.005 \times 0.8}{0.05375}$

= 0.0744

Answer: (a) 0.05375 (b) 0.0744

2 Exact values from the addition formulae (All boards)

By writing 15° as 45° − 30°, find the exact value of

sin 15°, in the form $\frac{\sqrt{a} \pm \sqrt{b}}{c}$.

sin 15° = sin(45° − 30°) = sin 45° cos 30° − cos 45° sin 30°

$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

Answer: $\frac{\sqrt{6} - \sqrt{2}}{4}$

3 Quadratic from its graph (All boards)

The diagram shows the curve with equation y = f(x), where f is a quadratic.
The turning point is marked, and the curve crosses the y-axis as shown.
Find f(x), giving it in the form $ax^2 + bx + c$.

Turning point (2, 6): $f(x) = a(x - 2)^2 + 6$
f(0) = 14: $a(2)^2 + 6 = 14$
a = 2, then expand

Answer: f(x) = $2x^2 - 8x + 14$

4 Connected particles (rough table, string slack) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (1 kg) lies on a rough table, μ = 0.25, joined by a light string over a smooth pulley to particle B (7 kg), 1.2 m above the floor. Released from rest; B lands without rebounding. Find (a) the acceleration of A before B lands, (b) the further distance A moves. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: 7 g − T = 7 a; A: T − 0.25×1 g = 1 a

Adding: (7 − 0.25×1)g = 8 a, so a = $\frac{1323}{160}$

Speed of B and A as B lands: $v^2 = 2 \times \frac{1323}{160} \times 1.2 = 19.845$

After B lands, A decelerates at μg = 0.25×9.8 = 2.45
$0 = v^2 - 2(\mu g)s$, s = 19.845 ÷ 4.9

Answer: (a) 8.27 m s^{-2} (b) 4.05 m

5 Transformations from a graph (All boards)

The diagram shows the curve y = f(x), with turning points A and B.
Find the coordinates of the turning points of
(a) y = 2f(x) (b) y = f(x) − 3

(a) y = 2f(x): A → (0, 12), B → (2, −4)
(b) y = f(x) − 3: A → (0, 3), B → (2, −5)

Answer: (a) (0, 12), (2, −4); (b) (0, 3), (2, −5)

6 Maximum speed from velocity function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with velocity $v = (-3t^2 + 12t)$ m s^{-1} at time t s, $t \geq 0$, starting at O. Find (a) its greatest speed, (b) its distance from O when t = 4, (c) the time at which it returns to O.

(a) $dv/dt = 12 - 6t = 0$, so $t = 2$ (a maximum since the t^2 coefficient is negative)
$v = 12\times2 - 3\times2^2 = 12$
(b) $x = \int v\,dt = 6\,t^2 - 1\,t^3$
$x(4) = 32$
(c) $x = 0$ when $t^2(6 - 1\,t) = 0$, so $t = 6$

Answer: (a) 12 m s^{-1} (b) 32 m (c) 6 s

7 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix}-2\\7\\7\end{pmatrix} + t\begin{pmatrix}2\\-2\\-2\end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(5, 4, 5) to l, and the exact distance from P to l.

Foot $F = A + t\,d$: $\mathrm{PF} = \begin{pmatrix}2t - 7\\-2t + 3\\-2t + 2\end{pmatrix}$
$\mathrm{PF}\cdot d = 0$ gives $12t - 24 = 0$, so $t = 2$
$F = (2, 3, 3)$; $\mathrm{PF} = \begin{pmatrix}-3\\-1\\-2\end{pmatrix}$, so $|\mathrm{PF}| = \sqrt{14}$

Answer: F(2, 3, 3), distance $\sqrt{14}$

8 Conditional normal probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(73, 4^2)$. Find $P(X > 81 \mid X > 75)$ to 3 significant figures.

Since $X > 81$ implies $X > 75$: $P = \dfrac{P(X > 81)}{P(X > 75)}$
$P(X > 81) = P(Z > 2.00) = 0.0228$
$P(X > 75) = P(Z > 0.500) = 0.309$
$P = \dfrac{0.0228}{0.309} = 0.0737$

Answer: 0.0737

9 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point (9, −8). Its centre lies in the fourth quadrant. Find the equations of the two possible circles.

Centre (r, −r), radius $r > 0$.
Through (9, −8): $(9 - r)^2 + (-8 + r)^2 = r^2$
$r^2 - 34r + 145 = 0$
$(r - 5)(r - 29) = 0$, so $r = 5$ or 29

Answer: $(x - 5)^2 + (y + 5)^2 = 25$ or $(x - 29)^2 + (y + 29)^2 = 841$

10 Area between curves and the x-axis (with sketch) (All boards)

The diagram shows the curve $y = x(x - 1)(x - 2)$.
Find the total area of the two shaded regions.

$y = x^3 - 3x^2 + 2x$; $\int y\,dx = \frac{1}{4}x^4 - x^3 + x^2$

From 0 to 1: $\frac{1}{4}$ (above the axis)

From 1 to 2: $-\frac{1}{4}$ (below the axis), so the area is $\frac{1}{4}$

Total = $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Answer: $\mathbf{\frac{1}{2}}$