Day 129 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix}-2\\2\end{pmatrix}$ and $\begin{pmatrix}-5\\-4\end{pmatrix}$. The point P lies on AB with AP : PB = 2 : 1.

Find the position vector of P and the exact distance OP.

$\text{OP} = \text{OA} + \frac{2}{3}\text{AB} = \begin{pmatrix}-2\\2\end{pmatrix} + \frac{2}{3}\begin{pmatrix}-3\\-6\end{pmatrix}$

$= \begin{pmatrix}-4\\-2\end{pmatrix}$

$\text{OP} = \sqrt{(-4)^2 + (-2)^2} = \sqrt{20} = 2\sqrt{5}$

Answer: $\begin{pmatrix}\mathbf{-4}\\\mathbf{-2}\end{pmatrix}$**, OP** $= \mathbf{2\sqrt{5}}$

2 Projected up a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 14 m s^{-1} up a line of greatest slope of a rough plane inclined at α, where $\tan\alpha = \frac{5}{12}$. The coefficient of friction is 0.25. Find (a) the greatest distance it moves up the plane, (b) its speed on returning to the start. Take g = 9.8.

Up: deceleration $= g(\sin\alpha + \mu\cos\alpha) = 9.8(\frac{5}{13} + 0.25\times\frac{12}{13}) = \frac{392}{65}$

(a) $0 = 14^2 - 2\times\frac{392}{65}\times d$, so d = 16.25

Down: $\tan\alpha = \frac{5}{12} > \mu$, so it slides back; acceleration $= g(\sin\alpha - \mu\cos\alpha) = \frac{98}{65}$

(b) $v^2 = 2\times\frac{98}{65}\times 16.25 = 49$

$v = \sqrt{49}$

Answer: (a) 16.25 m (b) 7 m s^{-1}

3 Venn diagrams and conditional probability (All boards)

P(A) = 0.35, P(B) = 0.75 and P(A ∩ B) = 0.25.

Find

(a) P(A ∪ B)

(b) P(B′ | A′)

(c) P(A | B)

(a) $\text{P}(\text{A}\cup\text{B}) = \text{P}(\text{A}) + \text{P}(\text{B}) - \text{P}(\text{A}\cap\text{B}) = 0.35 + 0.75 - 0.25 = 0.85$

(b) $\text{P}(\text{B}'\mid\text{A}') = \frac{\text{P}(\text{A}'\cap\text{B}')}{\text{P}(\text{A}')} = \frac{1-0.85}{1-0.35} = \frac{3}{13}$

(c) $\text{P}(\text{A}\mid\text{B}) = \frac{\text{P}(\text{A}\cap\text{B})}{\text{P}(\text{B})} = \frac{0.25}{0.75} = \frac{1}{3}$

Answer: (a) 0.85 (b) $\mathbf{\frac{3}{13}}$ **(c)** $\mathbf{\frac{1}{3}}$

4 Equations quadratic in disguise (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^4 - 14x^2 - 32 = 0$.

Let $u = x^2$: $u^2 - 14u - 32 = 0$

$(u - 16)(u + 2) = 0$ so u = 16 or u = −2

$x^2 = 16$ gives x = ±4

$x^2 = -2$ has no real solutions

Answer: x = ±4

5 Coding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 25 values of x, the coded variable $y = \frac{x - 30}{10}$ gives

$\Sigma y = 125$ and $\Sigma y^2 = 725$.

Find the mean and standard deviation of x.

$\bar{y} = \frac{125}{25} = 5$

$\sigma_y^2 = \frac{725}{25} - 5^2 = 4$, so $\sigma_y = 2$

$x = 10y + 30$, so mean $= 10 \times 5 + 30 = 80$

Standard deviation $= 10 \times 2 = 20$

Answer: mean 80, sd 20

6 Smooth incline held by a string (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 8 kg particle rests on a smooth plane inclined at α, $\tan\alpha = \frac{5}{12}$, held by a string in the vertical plane through a line of greatest slope, at angle θ above the slope, $\tan\theta = \frac{3}{4}$. Find the tension and the normal reaction. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$, $\sin\theta = 0.6$, $\cos\theta = 0.8$

Along the plane: $T\cos\theta = 8g\sin\alpha$, so $T = 37.7$

Perpendicular: $R + T\sin\theta = 8g\cos\alpha$

$R = 8g\cos\alpha - T\sin\theta = 72.4 - 37.7 \times 0.600$

Answer: T = 37.7 N, R = 49.8 N

7 Greatest and least values using R-form (All boards)

$g(x) = \frac{1}{18 + 15\sin x + 8\cos x}$

(a) Write $15\sin x + 8\cos x$ in the form $R\sin(x + \alpha)$, with α acute.

(b) Find the greatest and least values of g(x) and the smallest positive x at which the greatest value occurs (1 d.p.).

$R = \sqrt{15^2 + 8^2} = 17$, $\alpha = \tan^{-1}\frac{8}{15} = 28.1°$

Denominator $= 18 + 17\sin(x + 28.1°)$, ranging from 1 to 35

Greatest $g = \frac{1}{1}$ (denominator least), least $g = \frac{1}{35}$

Least denominator when $\sin(x + 28.1°) = -1$: $x + 28.1° = 270°$

$x = 241.9°$

Answer: R = 17, α = 28.1°; greatest $\frac{1}{1}$, least $\frac{1}{35}$; x = 241.9°

8 Recurrence relations: limits (All boards)

$u_1 = 18$ and $u_{n+1} = \frac{1}{3}u_n + 8$.

(a) Find the limit L of the sequence as $n \to \infty$.

(b) Find the least n for which $|u_n - L| < 0.01$.

(a) $L = \frac{1}{3}L + 8$ gives $L = \frac{8}{1 - \frac{1}{3}} = 12$

(b) $u_n - L = (u_1 - L)\,r^{n-1} = 6 \times \left(\frac{1}{3}\right)^{n-1}$

$|6| \times \left(\frac{1}{3}\right)^{n-1} < \frac{1}{100}$

Check: n = 6 gives 0.02469, n = 7 gives 0.00823

Answer: (a) L = 12 (b) n = 7

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9 Volumes of revolution (CIE (9709); Further Maths on other boards)

The region bounded by the curve $y = \frac{1}{x}$, the x-axis, $x = 1$ and $x = 3$ is rotated through 360° about the x-axis.

Find the exact volume of the solid formed.

$y^2 = \frac{1}{x^2}$, so $V = \pi\int_1^3 y^2\,dx$

$= \pi \times \frac{2}{3}$

$V = \frac{2\pi}{3}$

Answer: $\frac{2\pi}{3}$

10 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2t^2$, $y = -t(t^2 - 9)$.

Find the coordinates of the points where the curve meets the x-axis.

$y = 0$: $t = 0$ or $t^2 = 9$, so $t = 0, 3, -3$

$t = 0$: $x = 0$

$t = \pm 3$: $x = 18$

Answer: (0, 0) and (18, 0)