Day 128 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small x (in radians), use the small angle approximations to find the value that $\frac{1 - \cos 3x}{x \sin 4x}$ approaches as $x \to 0$.

$1 - \cos 3x \approx \frac{1}{2}(3x)^2 = \frac{9}{2}x^2$

$\sin 4x \approx 4x$

Expression $\approx \frac{\frac{9}{2}x^2}{x \times 4x} = \frac{9}{8} = \frac{9}{8}$

Answer: $\frac{9}{8}$

2 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean visibility (Dm) in the same month are:
Camborne: mean 25.5, standard deviation 5.4;
Leeming: mean 28, standard deviation 2.
Make two comparisons.

Means: Leeming has the higher mean (28 against 25.5), so on average the mean visibility is greater at Leeming
Standard deviations: Camborne is larger at Camborne (5.4 against 2)
so the readings at Camborne are more variable and those at Leeming are more consistent
Answer: Leeming higher on average; Camborne more variable

3 Inverse of a rational function (All boards)

$f(x) = \frac{-3x - 1}{x + 4}$, $x \in \mathbb{R}$, $x \neq -4$.

Find $f^{-1}(x)$ and state its domain.

$y = \frac{-3x - 1}{x + 4}$: $y(x + 4) = -3x - 1$

$xy + 4y = -3x - 1$

$x(y + 3) = -4y - 1$

$x = \frac{-4y - 1}{y + 3}$, then swap x and y

Answer: $f^{-1}(x) = \frac{-4x - 1}{x + 3}$, $x \neq -3$

4 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

$|a| = 4$, $|b| = 5$ and the angle between a and b is 150°.
Find the exact values of $a \cdot b$ and $|a + b|$.

$a \cdot b = |a||b|\cos 150° = 4 \times 5 \times -\frac{\sqrt{3}}{2} = -10\sqrt{3}$

$|a + b|^2 = |a|^2 + 2a \cdot b + |b|^2 = 16 + -20\sqrt{3} + 25 = 41 - 20\sqrt{3}$

$|a + b| = \sqrt{41 - 20\sqrt{3}}$

Answer: $a \cdot b = -10\sqrt{3}$, $|a + b| = \sqrt{41 - 20\sqrt{3}}$

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5 Projectile from and to the ground (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 31 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{12}{5}$, and lands at A. Find (a) the time of flight, (b) the greatest height, (c) OA. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component = $31\times\frac{5}{13} = \frac{155}{13}$; vertical component = $31\times\frac{12}{13} = \frac{372}{13}$

(a) Vertical: $0 = \frac{372}{13}t - 4.9t^2$, so $t = \frac{3720}{637}$

(b) At the top: $0 = \left(\frac{372}{13}\right)^2 - 2\times9.8\times H$, so $H = \frac{345960}{8281}$

(c) OA = $\frac{155}{13}\times\frac{3720}{637}$

Answer: (a) 5.84 s (b) 41.8 m (c) 69.6 m

6 Implicit differentiation: products and cubes (All boards)

A curve has equation $x^2y + 3y^3 + x = -9$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at (3, −1).

Differentiate: $2xy + x^2\frac{dy}{dx} + 9y^2\frac{dy}{dx} + 1 = 0$

$\frac{dy}{dx} = -\frac{2xy+1}{x^2+9y^2}$

At (3, −1): $\frac{dy}{dx} = \frac{5}{18}$, so the normal has gradient $-\frac{18}{5}$

Answer: $\frac{dy}{dx} = -\frac{2xy+1}{x^2+9y^2}$; normal gradient $-\frac{18}{5}$

7 Circle: tangent at a point (AQA, Edexcel, OCR, MEI, Eduqas)

The circle $x^2 + y^2 - 6x + 4y - 87 = 0$ passes through P(−3, −10).
(a) Find the equation of the tangent at P, in the form ax + by = c.
(b) Find the coordinates of the point where this tangent crosses the x-axis.

Centre (3, −2). Gradient of radius CP = $\frac{4}{3}$

Gradient of tangent = $-\frac{3}{4}$

$y + 10 = -\frac{3}{4}(x + 3)$: $3x + 4y = -49$

y = 0: $x = -\frac{49}{3}$

Answer: (a) 3x + 4y = −49 (b) $\left(-\frac{49}{3}, 0\right)$

8 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle α above the horizontal, $\tan\alpha = \frac{3}{4}$, and passes through A, 8 m horizontally from O and 3.5 m above the ground. Find the speed of projection. Take g = 9.8.

Let the speed be U. Horizontal: $8 = U\cos\alpha \times t$, so $t = 8 \div (0.8\,U)$
Vertical: $3.5 = U\sin\alpha \times t - 4.9t^2$
Eliminate t: $y = x\tan\alpha - 4.9x^2 \div (U^2\cos^2\alpha)$
$3.5 = 6 - 4.9\times8^2 \div (0.64\,U^2)$
$U^2 = 490 \div 2.5 = 196$, U = 14

Answer: 14 m s^{-1}

9 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Eggs are defective with probability 0.2, independently. A tray of 10 eggs is rejected if it has at least 3 defective. From 6 such trays, find the probability that exactly 2 are rejected (3 s.f.).

$Y \sim B(10, 0.2)$. $P(\text{rejected}) = P(Y \geq 3) = 1 - 0.678 = 0.322$

$W \sim B(6, 0.322)$ is the number rejected

$P(W = 2) = C(6, 2)(0.322)^2(1 - 0.322)^4 = 0.329$

Answer: 0.329

10 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{6} - 1) = -1 + 2\sqrt{6}$, writing x in the form $p + q\sqrt{6}$.

$$x = \frac{-1 + 2\sqrt{6}}{\sqrt{6} - 1}$$

Multiply top and bottom by $\sqrt{6} + 1$.

Numerator: $(-1 + 2\sqrt{6})(\sqrt{6} + 1) = 11 + \sqrt{6}$

Denominator: $6 - 1 = 5$

Answer: $x = \frac{11}{5} + \frac{1}{5}\sqrt{6}$