Day 127 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

For a sample of 6 pairs of values of time spent on social media and exam score, $r = -0.667$.
The critical value for a two-tailed test at the 5% level is 0.8114.
Test, at this level, whether there is any correlation between the two variables.

$H_0: \rho = 0$, $H_1: \rho \neq 0$
Critical value 0.8114
$|r| = 0.667 < 0.8114$
This is not in the critical region, so do not reject H_0.
There is insufficient evidence of correlation.
Answer: Do not reject H_0: insufficient evidence

2 Stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A driver at constant speed sees a hazard 50.4 m ahead. After a reaction time of 0.6 s she brakes with deceleration 8 m s^{-2} and stops exactly at the hazard. Find (a) her initial speed, (b) the total time from seeing the hazard to stopping.

Let the initial speed be v. Reaction distance = 0.6 v, braking distance = $v^2 \div (2\times8)$
$0.6\,v + v^2 \div 16 = 50.4$, so $v^2 + 9.6\,v - 806.4 = 0$
$(v - 24)(v + 33.6) = 0$, so $v = 24$
(b) Braking time = $24 \div 8 = 3$ s; total = 0.6 + 3
Answer: (a) 24 m s^{-1} (b) 3.6 s

3 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(−8, −3) and B(2, −3) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(−6, −7) lies on the circle.

Centre = midpoint of AB = (−3, −3)
$r^2 = (-5)^2 + (0)^2 = 25$
$(x + 3)^2 + (y + 3)^2 = 25$: $x^2 + y^2 + 6x + 6y - 7 = 0$
(b) $CM^2 = (-3)^2 + (-4)^2 = 25 = r^2$
Answer: (a) $x^2 + y^2 + 6x + 6y - 7 = 0$

4 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample is taken from a numbered list of 675 items.
The 4th item chosen is number 87 and the 5th item chosen is number 112.
Find the sampling interval, the number of the first item chosen and the size of the sample.

Interval = $(112 - 87) \div (5 - 4) = 25$
First item = $87 - 3 \times 25 = 12$
Sample size = $675 \div 25 = 27$
Answer: Interval 25, first 12, size 27

5 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 3 kg is acted on by forces $\mathbf{F_1} = (-14\mathbf{i} + 6\mathbf{j})$ N and $\mathbf{F_2}$. Its acceleration is $(-2\mathbf{i} - 3\mathbf{j})$ m s^{-2}. Find (a) $\mathbf{F_2}$, (b) the angle between $\mathbf{F_2}$ and $\mathbf{i}$, to 1 d.p.

$\mathbf{F} = m\mathbf{a}$: $\mathbf{F_1} + \mathbf{F_2} = 3(-2\mathbf{i} - 3\mathbf{j}) = -6\mathbf{i} - 9\mathbf{j}$
(a) $\mathbf{F_2} = (-6\mathbf{i} - 9\mathbf{j}) - (-14\mathbf{i} + 6\mathbf{j}) = 8\mathbf{i} - 15\mathbf{j}$
(b) $|\mathbf{F_2}| = \sqrt{(8^2 + -15^2)} = 17$
$\cos\theta = 8 \div 17$, so $\theta = 61.9°$
Answer: (a) $\mathbf{F_2} = (8\mathbf{i} - 15\mathbf{j})$ N (b) 61.9°

6 Equations with sin 2x and a trig function (All boards)

Solve $\sin 2\theta = \sqrt{3}\sin\theta$ for $-180° \le \theta \le 180°$.

$\sin 2\theta \equiv 2\sin\theta\cos\theta$: $\sin\theta(2\cos\theta - \sqrt{3}) = 0$

$\sin\theta = 0$: $\theta = -180°, 0°, 180°$

$\cos\theta = \frac{\sqrt{3}}{2}$: $\theta = -30°, 30°$

Do not divide by $\sin\theta$, as that would lose solutions

Answer: $\theta = -180°, -30°, 0°, 30°, 180°$

7 Circle: tangent at a point (AQA, Edexcel, OCR, MEI, Eduqas)

The circle $x^2 + y^2 - 2x + 4y - 20 = 0$ passes through P$(-2, 2)$.

(a) Find the equation of the tangent at P, in the form $ax + by = c$.

(b) Find the coordinates of the point where this tangent crosses the x-axis.

Centre $(1, -2)$. Gradient of radius CP $= -\frac{4}{3}$

Gradient of tangent $= \frac{3}{4}$

$y - 2 = \frac{3}{4}(x + 2)$: $3x - 4y = -14$

$y = 0$: $x = -\frac{14}{3}$

Answer: (a) $3x - 4y = -14$ (b) $\left(-\frac{14}{3}, 0\right)$

8 Optimisation problems (All boards)

An open-topped box has a square base of side x cm and volume 500 cm^3.

Show that its surface area is $A = x^2 + \frac{2000}{x}$, and find the least possible value of A, justifying that it is a minimum.

$h = \frac{500}{x^2}$, so $A = x^2 + 4xh = x^2 + \frac{2000}{x}$

$\frac{dA}{dx} = 2x - \frac{2000}{x^2} = 0$, so $x^3 = 1000$ and $x = 10$

$\frac{d^2A}{dx^2} = 2 + \frac{4000}{x^3} > 0$, so a minimum

$A = 100 + \frac{2000}{10} = 300$

Answer: $A_{min} = 300\ cm^2$ ($x = 10$)

9 Tangent conditions (All boards)

The line $y = kx - 3$ is a tangent to the curve $y = x^2 - 6x + 6$.

(a) Find the possible values of k.

(b) For the larger value of k, find the coordinates of the point of contact.

Equate: $x^2 - 6x + 6 = kx - 3$

$x^2 + (-6 - k)x + 9 = 0$

Tangent: discriminant = 0: $(-6 - k)^2 = 36$

$-6 - k = \pm 6$ so $k = 0$ or -12

$k = 0$: $x = \frac{k - (-6)}{2} = 3$, $y = -3$

Answer: (a) $k = -12$ or $k = 0$ (b) $(3, -3)$

10 Volumes of revolution (CIE (9709); Further Maths on other boards)

The region bounded by the curve $y = x^2 + 1$, the x-axis, $x = 0$ and $x = 2$ is rotated through 360° about the x-axis. Find the exact volume of the solid formed.

$y^2 = (x^2 + 1)^2 = x^4 + 2x^2 + 1$, so $V = \pi\int_0^2 y^2\,dx$

$= \pi \times \frac{206}{15}$

$V = \frac{206\pi}{15}$

Answer: $\mathbf{\frac{206\pi}{15}}$