Day 126 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Chain rule and gradients (All boards)

$y = (\ln x)^2$ for $x > 0$.

Find $\frac{dy}{dx}$ and its exact value at $x = e^3$.

$\frac{dy}{dx} = 2 \ln x \times \frac{1}{x} = \frac{2 \ln x}{x}$

At $x = e^3$: $\ln x = 3$, so gradient $= \frac{6}{e^3}$

Answer: $\frac{dy}{dx} = \frac{2 \ln x}{x}$; **gradient** $= \frac{6}{e^3}$

2 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A club has 200 members: 32 Bronze, 104 Silver and 64 Gold.
A stratified sample of size 25 is taken.
Find the number of members sampled from each group.

Sampling fraction $= \frac{25}{200} = \frac{1}{8}$

Bronze: $32 \times \frac{1}{8} = 4$

Silver: $104 \times \frac{1}{8} = 13$

Gold: $64 \times \frac{1}{8} = 8$

Answer: Bronze 4, Silver 13, Gold 8

3 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^4 in the expansion of

$(3x - \frac{1}{x})^6$.

General term: ${}^6C_r \, (3x^1)^{6-r} \, (-x^{-1})^r$
Power of x: $1(6 - r) - 1r = 4$
$6 - 2r = 4$ so $r = 1$
Coefficient $= {}^6C_1 \times 3^5 \times (-1)^1 = -1458$
Answer: -1458

4 Fixed point iteration (All boards)

The equation $x^3 = 6x + 2$ has a root α. Use the iteration $x_{n+1} = \sqrt[3]{6x + 2}$ with $x_0 = 3$
to find x_1, x_2 and x_3 to 4 d.p., and hence the value of α to 2 d.p. (continue until the 2 d.p. value is settled).

$x_1 = \sqrt[3]{6 \times 3 + 2} = 2.7144$
$x_2 = 2.6346$, $x_3 = 2.6114$
Continuing, the values settle at 2.6017, so $\alpha = 2.60$
Answer: $x_1 = 2.7144$, $x_2 = 2.6346$, $x_3 = 2.6114$; $\alpha = 2.60$

5 Correcting a recorded value (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

15 values have $\Sigma x = 885$ and $\Sigma x^2 = 52950$.
One value was recorded as 52 but should have been 55.
Find the correct mean and standard deviation (3 s.f.).

$\Sigma x = 885 - 52 + 55 = 888$
$\Sigma x^2 = 52950 - 52^2 + 55^2 = 53271$

Mean $= \frac{888}{15} = 59.2$

sd $= \sqrt{\frac{53271}{15} - (59.2)^2} = 6.84$

Answer: mean 59.2, sd 6.84

6 Volumes of revolution (CIE (9709); Further Maths on other boards)

The region bounded by the curve $y = \sin x$, the x-axis, $x = 0$ and $x = \pi$ is rotated through 360° about the x-axis. Find the exact volume of the solid formed.

$y^2 = \sin^2 x = \frac{1}{2}(1 - \cos 2x)$, so $V = \pi\int_0^{\pi} y^2\,dx$

$= \pi \times \frac{\pi}{2}$

$V = \frac{\pi^2}{2}$

Answer: $\boldsymbol{\frac{\pi^2}{2}}$

7 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 3t^2 - 1$, $y = -2t(t^2 - 9)$. Find the coordinates of the points where the curve meets the x-axis.

$y = 0$: $t = 0$ or $t^2 = 9$, so $t = 0, 3, -3$
$t = 0$: $x = -1$
$t = \pm 3$: $x = 26$
Answer: (−1, 0) and (26, 0)

8 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (3 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (1 kg), hanging 0.5 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

B: $1\,g - T = 1\,a$
A: $T = 3\,a$
Adding: $1\,g = 4\,a$, so $a = 2.45$
(a) $T = 3 \times 2.45 = 7.35$
(b) $v^2 = 0 + 2 \times 2.45 \times 0.5 = 2.45$
Answer: (a) 7.35 N (b) 1.57 m s^{-1}

9 Catching up (delayed start) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes P at constant speed 4 m s^{-1}. 2 s later car B leaves P from rest, in the same direction, with acceleration 3 m s^{-2}. B overtakes A at Q. Find (a) the time B takes to reach Q, (b) PQ, (c) the speed of B at Q.

Let B take s seconds; A has then been moving for (s + 2) seconds.
B: $\frac{1}{2} \times 3 \times s^2 = 1.5\,s^2$ A: $4(s + 2)$
$1.5\,s^2 - 4\,s - 8 = 0$ so $s = 4$ ($s > 0$)
(b) $PQ = 1.5 \times 4^2 = 24$
(c) $v = 0 + at = 3 \times 4 = 12$
Answer: (a) 4 s (b) 24 m (c) 12 m s^{-1}

10 Equation of a cubic from its graph (All boards)

The diagram shows a sketch of the curve $y = f(x)$, where $f(x)$ is a cubic. Find $f(x)$, giving your answer in expanded form.

Roots at $x = -3, -1, 3$, so $f(x) = k(x + 3)(x + 1)(x - 3)$
$f(0) = -27$: $k(3)(1)(-3) = -27$
$k = 3$, then expand
Answer: $\mathbf{f(x) = 3x^3 + 3x^2 - 27x - 27}$