Day 124 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Tree diagram with conditional probability (All boards)

The tree diagram shows probabilities for events A and B.

(a) Find P(B).

(b) Find P(A | B).

(a) $P(B) = 0.5 \times 0.8 + 0.5 \times 0.9 = 0.85$

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.5 \times 0.8}{0.85}$

Answer: (a) 0.85 (b) $\frac{8}{17}$

2 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{2}} (\sin 2x + 4\cos x)\,dx$.

Integrate: $[-\frac{1}{2}\cos 2x + 4\sin x]$

Substitute the limits and subtract: 5

Answer: 5

3 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $x^2 - 12x + 38 > 0$ for all real values of x.

Complete the square: $x^2 - 12x + 38 = (x - 6)^2 + 2$

$(x - 6)^2 \geq 0$ for all x, so the expression is $\geq 2 > 0$.

Answer: $(x - 6)^2 + 2 \geq 2 > 0$

4 One-tailed binomial hypothesis test (AQA, Edexcel, OCR, MEI, Eduqas)

A claim is that the probability p that a spinner lands on red is 0.5. In a sample of 12, the number of successes is 5. Test at the 5% level whether p has decreased.

H_0: $p = 0.5$, H_1: $p < 0.5$; $X \sim B(12, 0.5)$

$P(X \leq 5) = 0.387$

$0.387 > 0.05$

Do not reject H_0: insufficient evidence of a change in p.

Answer: p-value 0.387; do not reject H_0

5 Completing the square (All boards)

$f(x) = 2x^2 + 16x + 26$.

(a) Write f(x) in the form $a(x + p)^2 + q$.

(b) State the minimum value of f(x) and the value of x at which it occurs.

(c) Solve $f(x) = 0$, giving surds in simplest form.

$f(x) = 2(x + 4)^2 - 6$

(b) minimum value -6 when $x = -4$

(c) $2(x + 4)^2 = 6$, so $(x + 4)^2 = 3$

$x + 4 = \pm\sqrt{3}$

Answer: (a) $2(x + 4)^2 - 6$ (b) minimum -6 at $x = -4$ (c) $x = -4 \pm \sqrt{3}$

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6 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
$\sin\theta + 2\cos 3\theta = 0.5$
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin\theta \approx \theta$, $\cos 3\theta \approx 1 - \frac{9}{2}\theta^2$

$1 \times \theta + 2(1 - \frac{9}{2}\theta^2) = 0.5$

Multiply by 2 and rearrange: $18\theta^2 - 2\theta - 3 = 0$

$\theta = \frac{2 \pm \sqrt{220}}{36}$, so $\theta = 0.468$

Answer: a = 18, b = −2, c = −3; $\theta \approx 0.468$

7 Sum to infinity: two possible ratios (All boards)

The second term of a geometric series is 4 and its sum to infinity is 25.
Find the two possible values of the first term.

$ar = 4$ and $\frac{a}{(1 - r)} = 25$

$a = 25(1 - r)$, so $25(1 - r)\, r = 4$

$25r^2 - 25r + 4 = 0$: $r = \frac{1}{5}$ or $r = \frac{4}{5}$

$a = 25(1 - r) = 20$ or 5

Answer: a = 5 or a = 20

8 Catching up (delayed start) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes P at constant speed 10 m s^{-1}. 10 s later car B leaves P from rest, in the same direction, with acceleration 1.5 m s^{-2}. B overtakes A at Q. Find (a) the time B takes to reach Q, (b) PQ, (c) the speed of B at Q.

Let B take s seconds; A has then been moving for (s + 10) seconds.
B: $\frac{1}{2}\times 1.5\times s^2 = 0.75\, s^2$ A: $10(s + 10)$
$0.75\, s^2 - 10\, s - 100 = 0$ so $s = 20$ ($s > 0$)
(b) $PQ = 0.75\times 20^2 = 300$
(c) $v = 0 + at = 1.5\times 20 = 30$

Answer: (a) 20 s (b) 300 m (c) 30 m s^{-1}

9 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $-x^2 + 4xy - 4x = -12$.
Show that P(−2, 2) lies on the curve and find the equation of the tangent at P in the form $ax + by + c = 0$ with integer a, b, c.

Differentiate (product rule on the xy term): $(-2x + 4y - 4) + (4x)\frac{dy}{dx} = 0$

At P(−2, 2): $8 + (-8)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = 1$

Tangent: $y - 2 = 1(x + 2)$, i.e. $x - y + 4 = 0$

Answer: x − y + 4 = 0

10 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 12 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{12}{5}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity $(\frac{60}{13}, \frac{144}{13})$. At time T the velocity is $(\frac{60}{13}, \frac{144}{13} - 9.8T)$.

Perpendicular: scalar product = 0: $(\frac{60}{13})^2 + \frac{144}{13}(\frac{144}{13} - 9.8T) = 0$

$12^2 - 9.8T \times \frac{144}{13} = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 12 \div (9.8 \times \frac{12}{13}) = \frac{65}{49}$

(b) $h = \frac{144}{13} \times \frac{65}{49} - 4.9 \times \frac{65}{49}^2 = \frac{85}{14}$

Answer: (a) T = 1.33 s (b) 6.07 m