Day 123 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Laws of logarithms in terms of p and q (All boards)

Given that $\log_a 2 = p$ and $\log_a 3 = q$,

express $\log_a\left(\frac{\sqrt{8}}{\sqrt{27}}\right)$ in terms of p and q.

$\frac{\sqrt{8}}{\sqrt{27}} = 2^{\frac{3}{2}} \times 3^{-\frac{3}{2}}$

$\log_a(2^{\frac{3}{2}} \times 3^{-\frac{3}{2}}) = \frac{3}{2}\log_a 2 - \frac{3}{2}\log_a 3$

$= \frac{3}{2}p - \frac{3}{2}q$

Answer: $\frac{3}{2}p - \frac{3}{2}q$

2 Transformations of a turning point (All boards)

The curve y = f(x) has a minimum point at (5, −6).
Find the coordinates of the corresponding point on:
(a) y = 4f(x)
(b) y = f(x) + 1
(c) y = f(2x)

(a) y = 4f(x): (5, −24)
(b) y = f(x) + 1: (5, −5)
(c) y = f(2x): $(\frac{5}{2}, -6)$

A stretch or translation moves the point in the stated direction only.

Answer: (a) (5, −24); (b) (5, −5); (c) $(\frac{5}{2}, -6)$

3 Beam on two supports (AQA, Edexcel, OCR, MEI)

A uniform beam AB rests on supports at C and D, with a particle attached at A. Find the reactions at C and D. Take g = 9.8.

Moments about C: $R_D \times 7 = 25\,g \times 3.5 - 2\,g \times 1.5$
$R_D = 118$
Resolving vertically: $R_C + R_D = 27\,g = 264.6$
$R_C = 264.6 - 118$
Answer: R_C = 146.3 N, R_D = 118.3 N

4 Sigma notation (All boards)

Evaluate $\sum_{r=9}^{31}(2r - 1)$.

Arithmetic series: first term 17, last term 61, 23 terms
$S = \frac{23}{2}(17 + 61) = 897$

Answer: 897

5 Double angle equations (All boards)

Solve $\cos 2x - 3\sin x = -1$
for $0° \le x \le 360°$.

$\cos 2x \equiv 1 - 2\sin^2 x$: $2\sin^2 x + 3\sin x - 2 = 0$

Solve the quadratic: $\sin x = \frac{1}{2}$ or $\sin x = -2$

$\sin x = \frac{1}{2}$: x = 30°, 150°

$\sin x = -2$ is impossible

Answer: x = 30°, 150°

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6 Normal approximation to the binomial (All boards)

X ~ B(80, 0.4). Use a normal approximation to estimate P(X ≥ 34) to 3 significant figures.

np = 32, npq = 19.2, so Y ~ N(32, 19.2)
With continuity correction: P(Y > 33.5)
$z = \frac{33.5 - 32}{4.38} = 0.342$, P(Z > 0.342) = 0.366

Answer: 0.366

7 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small x (in radians), use the small angle approximations to find the value that $\frac{1 - \cos 2x}{x \sin x}$ approaches as $x \to 0$.

$1 - \cos 2x \approx \frac{1}{2}(2x)^2 = \frac{4}{2}x^2$

$\sin x \approx x$

Expression $\approx \frac{\frac{4}{2}x^2}{x \times x} = \frac{4}{2} = 2$

Answer: 2

8 Range of a composite function (All boards)

$f(x) = x^2 - 4x + 8$, $x \in \mathbb{R}$, and $g(x) = \frac{6}{x + 1}$, $x > -1$.

(a) Find the range of f.
(b) Show that gf is defined and find its range.

$f(x) = (x - 2)^2 + 4 \geq 4$
So $f(x) \geq 4 > -1$, which lies in the domain of g, so gf is defined.

$gf(x) = \frac{6}{f(x) + 1}$, with $f(x) \geq 4$, so $f(x) + 1 \geq 5$

Largest value is $\frac{6}{5} = \frac{6}{5}$ at the minimum of f; gf(x) > 0.

Answer: (a) $f(x) \geq 4$ (b) $0 < gf(x) \leq \frac{6}{5}$

9 Bayes' theorem in context (All boards)

5% of people have a disease. The screening test is positive for 90% of those who have it and for 5% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the disease (3 s.f.).

(a) P(+) = 0.05 × 0.9 + 0.95 × 0.05
= 0.045 + 0.0475 = 0.0925

(b) $P(\text{has it} \mid +) = \frac{0.05 \times 0.9}{0.0925}$

= 0.486

Answer: (a) 0.0925 (b) 0.486

10 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle α above the horizontal, $\tan \alpha = \frac{3}{4}$, and passes through A, 48 m horizontally from O and 13.5 m above the ground. Find the speed of projection. Take g = 9.8.

Let the speed be U. Horizontal: 48 = U cos α × t, so t = 48 ÷ (0.8 U)
Vertical: $13.5 = U \sin \alpha \times t - 4.9t^2$
Eliminate t: $y = x \tan \alpha - 4.9x^2 \div (U^2 \cos^2 \alpha)$
$13.5 = 36 - 4.9 \times 48^2 \div (0.64\ U^2)$
$U^2 = 17640 \div 22.5 = 784$, U = 28

Answer: 28 m s^{-1}