**Day 122** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Quadratic from its graph (All boards)

The diagram shows the curve with equation y = f(x), where f is a quadratic.
The turning point is marked, and the curve crosses the y-axis as shown.
Find f(x), giving it in the form $ax^2 + bx + c$.

Turning point (2, −2): $f(x) = a(x - 2)^2 - 2$
$f(0) = -14$: $a(2)^2 - 2 = -14$
$a = -3$, then expand
**Answer: $f(x) = -3x^2 + 12x - 14$**

## 2 Exact definite integrals of standard functions (All boards)

Find the exact value of

$$\int_0^{\ln 4} (4e^{2x} + e^x)\,dx.$$

Integrate: $[2e^{2x} + e^x]$
Substitute the limits and subtract: 33
**Answer: 33**

## 3 Venn diagrams and conditional probability (All boards)

P(A) = 0.6, P(B) = 0.4 and P(A ∩ B) = 0.2.
Find
(a) P(A ∪ B)
(b) P(A | B′)
(c) P(B | A)

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.4 - 0.2 = 0.8$
(b) $P(A \mid B') = \frac{P(A \cap B')}{P(B')} = \frac{0.6 - 0.2}{1 - 0.4} = \frac{2}{3}$
(c) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.6} = \frac{1}{3}$
**Answer: (a) 0.8 (b) $\frac{2}{3}$ (c) $\frac{1}{3}$**

## 4 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that the sum of the squares of any two consecutive integers is odd.

Let the integers be n and n + 1.
$n^2 + (n + 1)^2 = 2n^2 + 2n + 1 = 2(n^2 + n) + 1$
$n^2 + n$ is an integer, so this is 2 × integer + 1.
**Answer: $2(n^2 + n) + 1$, which is odd**

## 5 Equations with sin 2x and a trig function (All boards)

Solve sin 2x = sin x for $-180° \le x \le 180°$.

$\sin 2x \equiv 2\sin x \cos x$: $\sin x (2\cos x - 1) = 0$
$\sin x = 0$: $x = -180°, 0°, 180°$

$\cos x = \frac{1}{2}$: $x = -60°, 60°$

Do not divide by sin x, as that would lose solutions
**Answer: $x = -180°, -60°, 0°, 60°, 180°$**

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## 6 Periodic sequences (All boards)

A sequence is defined by $u_1 = \frac{5}{2}$ and $u_{n+1} = 1 - \frac{1}{u_n}$.

(a) Find $u_2$ and $u_3$.

(b) Find $u_{99}$ and $\sum_{r=1}^{99} u_r$.

$u_2 = \frac{3}{5}$, $u_3 = -\frac{2}{3}$, $u_4 = \frac{5}{2}$

$u_4 = u_1$, so the sequence repeats with period 3.

$99 = 3 \times 33 + 0$, so $u_{99} = u_3 = -\frac{2}{3}$

Sum of one period = $\frac{73}{30}$; 33 full periods plus 0 extra term(s)

**Answer: (a) $\frac{3}{5}$, $-\frac{2}{3}$ (b) $u_{99} = -\frac{2}{3}$, sum = $\frac{803}{10}$**

## 7 Equations with cosec x and cot x (All boards)

Solve $3\text{cosec}^2\,\theta - 15\cot\theta + 15 = 0$
for $-180° < \theta < 180°$, $\theta \neq 0$. Give answers to 1 d.p.

Use $\text{cosec}^2\,\theta \equiv 1 + \cot^2\theta$: $3\cot^2\theta - 15\cot\theta + 18 = 0$
$\cot\theta = 3$ or $\cot\theta = 2$

$\cot\theta = 3$ means $\tan\theta = \frac{1}{3}$: $\theta = -161.6°, 18.4°$

$\cot\theta = 2$ means $\tan\theta = \frac{1}{2}$: $\theta = -153.4°, 26.6°$

**Answer: $\theta = -161.6°, -153.4°, 18.4°, 26.6°$**

## 8 Projectile from a cliff top (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a cliff top 7.6 m above the sea at 10 m $s^{-1}$, at angle $\alpha$ above the horizontal, $\tan\alpha = \frac{3}{4}$. Find (a) the time to reach the sea, (b) the horizontal distance travelled, (c) the speed on impact. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component 8, vertical component 6 (upwards)
(a) $-7.6 = 6t - 4.9t^2$, so $4.9t^2 - 6t - 7.6 = 0$
$t = [6 + \sqrt{184.96}] \div 9.8 = 2$ (taking the positive root)
(b) Distance = 8×2
(c) Vertical speed = $6 - 9.8\times2 = -13.6$; speed = $\sqrt{(8^2 + 13.6^2)}$

**Answer: (a) 2 s (b) 16 m (c) 15.8 m $s^{-1}$**

## 9 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(28, p)$. $H_0$: $p = 0.3$ is tested against $H_1$: $p < 0.3$ using a 5% significance level.
(a) Find the critical region. (b) State the probability of a Type I error.
(c) Find the probability of a Type II error when $p = 0.1$ (3 s.f.).

(a) Largest/smallest region with probability $\leq 0.05$ under $H_0$: $X \leq 4$
(b) Type I error = P(reject $H_0$ | $p = 0.3$) = 0.0474
(c) Type II error = P(not in critical region | $p = 0.1$)
$P(X \geq 5)$ when $p = 0.1$: 0.142

**Answer: (a) $X \leq 4$ (b) 0.0474 (c) 0.142**

## 10 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 38 m $s^{-1}$ at angle $\alpha$ above the horizontal, $\tan\alpha = \frac{24}{7}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity (10.64, 36.48). At time T the velocity is (10.64, 36.48 − 9.8T).
Perpendicular: scalar product = 0: $(10.64)^2 + 36.48(36.48 - 9.8T) = 0$
$38^2 - 9.8T \times 36.48 = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 38 \div (9.8 \times 0.96) = \frac{2375}{588}$

(b) $h = 36.48 \times \frac{2375}{588} - 4.9 \times \left(\frac{2375}{588}\right)^2 = \frac{951235}{14112}$

**Answer: (a) T = 4.04 s (b) 67.4 m**