Day 120 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Change of sign to locate a root (All boards)

$f(x) = \ln x + 4x - 7$.
Show that the equation $f(x) = 0$ has a root α in the interval $[1, 2]$.

$f(x)$ is continuous on the interval
$f(1) = -3.00 < 0$
$f(2) = 1.69 > 0$
A change of sign, so a root lies in $[1, 2]$
Answer: Sign change: $f(1) = -3.00$, $f(2) = 1.69$

2 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 250 kg carries a person of mass 40 kg. The cable tension is 2610 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take $g = 9.8$.

Whole system (mass 290 kg), taking upwards as positive: $T - 290\,g = 290\,a$
$2610 - 2842 = 290\,a$, so $a = -0.8$
(a) a is negative, so the acceleration is 0.8 m s^{-2} downwards
(b) Person: $R - 40\,g = 40\,a$, so $R = 40(9.8 - 0.8)$
Answer: (a) 0.8 m s^{-2} downwards (b) 360 N

3 Interpolation for median and percentiles (All boards)

The table summarises the journey time, t minutes of 79 items.
Use linear interpolation to estimate the 90th percentile.

Position $= 0.9 \times 79 = 71.1$
Cumulative frequency before the class $100 \leq t < 120$ is 70
Estimate $= 100 + \frac{71.1 - 70}{9} \times 20$
$= 102$
Answer: 102

4 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$. The point P lies on AB with AP : PB = 2 : 1.
Find the position vector of P and the exact distance OP.

$\overrightarrow{OP} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} -3 \\ -3 \end{pmatrix}$
$= \begin{pmatrix} 4 \\ -4 \end{pmatrix}$
$OP = \sqrt{4^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$
Answer: $\begin{pmatrix} \mathbf{4} \\ \mathbf{-4} \end{pmatrix}$, OP = $4\sqrt{2}$

5 Outliers (AQA, Edexcel, OCR, MEI, Eduqas)

A data set has mean 53 and standard deviation 4.
A value is treated as an outlier if it is more than 3 standard deviations from the mean.
Which of 38, 63, 68, 73 are outliers?

3 standard deviations $= 3 \times 4 = 12$
Lower limit $= 53 - 12 = 41$
Upper limit $= 53 + 12 = 65$
Outliers are values below 41 or above 65
Answer: 38, 68, 73

6 Arithmetic series: S_n formula (All boards)

The sum of the first n terms of an arithmetic series is $S_n = n^2 + n$.

(a) Find the first term and the common difference.

(b) Find u_{28}.

$u_1 = S_1 = 2$

$S_2 = 6$, so $u_2 = 4$

$d = u_2 - u_1 = 2$

$u_{28} = 2 + 27 \times 2$

Answer: (a) a = 2, d = 2 (b) 56

7 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} -8 \\ -5 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ -3 \\ 3 \end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(−3, 1, −4) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathrm{PF} = \begin{pmatrix} -3t - 5 \\ -3t - 6 \\ 3t + 7 \end{pmatrix}$

PF·d = 0 gives 27t + 54 = 0, so t = −2

F = (−2, 1, −3); $\mathrm{PF} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, so $|\mathrm{PF}| = \sqrt{2}$

Answer: F(−2, 1, −3), distance $\sqrt{2}$

8 Modulus graph and a line (All boards)

The diagram shows the graphs of $y = |2x - 1|$ and $y = -x + 5$.

The graphs meet at A and B.

Find the coordinates of A and B.

$|2x - 1| = -x + 5$

Solve $2x - 1 = \pm(-x + 5)$.

x = −4 and x = 2 (both checked in the original equation)

y = 9 and y = 3

Answer: A(−4, 9), B(2, 3)

9 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $4x - 3y = 37$ meets the circle $(x - 4)^2 + (y + 1)^2 = 49$ at the points A and B. Find the exact length of AB.

Centre (4, −1), radius 7. Distance from centre to line:

$d = \frac{|-18|}{5} = \frac{18}{5}$

$\text{Half-chord}^2 = 49 - (\frac{18}{5})^2 = \frac{901}{25}$

$\mathrm{AB} = 2\sqrt{\frac{901}{25}}$

Answer: $\frac{10\sqrt{901}}{25}$

10 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 3 kg particle hangs in equilibrium from two light strings, as shown. Find the tension in each string (T_1 in the string at angle α). Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$, $\sin\beta = \frac{12}{13}$, $\cos\beta = \frac{5}{13}$

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_2 = 2.4\ T_1$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 3\ g = 29.4$

$T_1(\frac{5}{13} + \frac{144}{65}) = 29.4$

$T_1 = 11.3$, $T_2 = 27.1$

Answer: T_1 = 11.3 N, T_2 = 27.1 N