Day 119 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Double angle equations (All boards)

Solve $\cos 2x + 4\cos x = 5$
for $0° \leq x \leq 360°$.

$\cos 2x \equiv 2\cos^2 x - 1$: $2\cos^2 x + 4\cos x - 6 = 0$
Solve the quadratic: $\cos x = 1$ or $\cos x = -3$
$\cos x = 1$: $x = 0°, 360°$
$\cos x = -3$ is impossible
Answer: $x = 0°, 360°$

2 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 6 kg particle rests on a rough plane inclined at α, $\tan\alpha = \frac{24}{7}$, $\mu = 0.2$. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

$\sin\alpha = 0.96$, $\cos\alpha = 0.28$, $R = 6g\cos\alpha = 16.464$ N
Limiting friction $= \mu R = 3.2928$ N
P smallest: friction acts up the slope: $P + \mu R = 6g\sin\alpha$, $P = 56.448 - 3.2928$
P greatest: friction acts down the slope: $P = 56.448 + 3.2928$
Answer: $53.2 \text{ N} \leq P \leq 59.7 \text{ N}$

3 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample is taken from a numbered list of 285 items.
The 5th item chosen is number 72 and the 7th item chosen is number 102.
Find the sampling interval, the number of the first item chosen and the size of the sample.

Interval $= (102 - 72) \div (7 - 5) = 15$
First item $= 72 - 4 \times 15 = 12$
Sample size $= 285 \div 15 = 19$
Answer: Interval 15, first 12, size 19

4 Final seconds of a fall (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a cliff. In the last 2 seconds before it hits the sea it falls 78.4 m. Find the height of the cliff. Take $g = 9.8$.

Let the total time of fall be t s. Then $s = 4.9t^2$ for the whole fall.
In the first $(t - 2)$ s the stone falls $4.9(t - 2)^2$.
$4.9t^2 - 4.9(t - 2)^2 = 78.4$
$4.9(4t - 4) = 78.4$, so $t = 5$
Height $= 4.9 \times 5^2 = 122.5$
Answer: 122.5 m

5 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = 2 + 3\arccos(2x)$.
Find the exact value of x for which $f(x) = \pi + 2$.
Hence state the range of f.

$3\arccos(2x) = \pi$, so $\arccos(2x) = \frac{\pi}{3}$

$2x = \cos\frac{\pi}{3} = \frac{1}{2}$, so $x = \frac{1}{4}$

arccos takes values from 0 to π, so f(x) runs from 2 to $3\pi + 2$

Answer: $x = \frac{1}{4}$; range $2 \leq f(x) \leq 3\pi + 2$

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6 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{7} - 3) = 5 + 4\sqrt{7}$, writing x in the form $p + q\sqrt{7}$.

$x = \dfrac{5 + 4\sqrt{7}}{\sqrt{7} - 3}$

Multiply top and bottom by $\sqrt{7} + 3$.

Numerator: $(5 + 4\sqrt{7})(\sqrt{7} + 3) = 43 + 17\sqrt{7}$

Denominator: $7 - 9 = -2$

Answer: $\mathbf{x = -\frac{43}{2} - \frac{17}{2}\sqrt{7}}$

7 Volumes of revolution (CIE (9709); Further Maths on other boards)

The region bounded by the curve $y = e^{3x}$, the x-axis, $x = 0$ and $x = 1$ is rotated through 360° about the x-axis. Find the exact volume of the solid formed.

$y^2 = e^{6x}$, so $V = \pi\int_0^1 y^2\,dx$

$= \pi \times \dfrac{e^6 - 1}{6}$

$V = \dfrac{\pi e^6 - \pi}{6}$

Answer: $\mathbf{\dfrac{\pi e^6 - \pi}{6}}$

8 Line and curve intersection (diagram) (All boards)

The diagram shows the curve $y = x^2 - 2x - 2$ and the line $y = -2x + 2$, meeting at A and B.

(a) Find the coordinates of A and B.

(b) Find the coordinates of the midpoint of AB.

$x^2 - 2x - 2 = -2x + 2$: $x^2 - 4 = 0$

$(x + 2)(x - 2) = 0$, $x = -2$ or 2

A(−2, 6), B(2, −2)

Midpoint: average the coordinates

Answer: (a) A(−2, 6), B(2, −2) (b) (0, 2)

9 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A stratified sample of 80 students is taken from three groups.

5 of the sample are from Arts, which has 40 students.

(a) Find the total number of students.

(b) Science has 96 students. How many are sampled?

Arts sampled fraction = $\dfrac{5}{40} = \dfrac{1}{8}$

Total = $80 \div \dfrac{1}{8} = 640$

Science: $96 \times \dfrac{1}{8} = 12$

Answer: (a) 640 (b) 12

10 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point (−5, 10). Its centre lies in the second quadrant. Find the equations of the two possible circles.

Centre (−r, r), radius $r > 0$.

Through (−5, 10): $(-5 + r)^2 + (10 - r)^2 = r^2$

$r^2 - 30r + 125 = 0$

$(r - 5)(r - 25) = 0$, so r = 5 or 25

Answer: $\mathbf{(x + 5)^2 + (y - 5)^2 = 25}$ **or** $\mathbf{(x + 25)^2 + (y - 25)^2 = 625}$