Day 118 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 5 m s^{-1} and accelerates uniformly. She travels 75 m in the next 10 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $75 = 5\times10 + \frac{1}{2}\times a\times10^2$, so $a = 0.5$
(b) $v = u + at = 5 + 0.5\times10 = 10$
(c) For the midpoint, $s = 37.5$: $v^2 = u^2 + 2as = 5^2 + 2\times0.5\times37.5$
$v^2 = 62.5$

Answer: (a) 0.5 m s^{-2} (b) 10 m s^{-1} (c) 7.91 m s^{-1}

2 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 3t^2 - 5$, $y = t(t^2 - 4)$.
Find the coordinates of the points where the curve meets the x-axis.

$y = 0$: $t = 0$ or $t^2 = 4$, so $t = 0, 2, -2$
$t = 0$: $x = -5$
$t = \pm2$: $x = 7$

Answer: (−5, 0) and (7, 0)

3 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 250 kg carries a person of mass 70 kg. The cable tension is 2880 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take $g = 9.8$.

Whole system (mass 320 kg), taking upwards as positive: $T - 320g = 320a$
$2880 - 3136 = 320a$, so $a = -0.8$
(a) a is negative, so the acceleration is 0.8 m s^{-2} downwards
(b) Person: $R - 70g = 70a$, so $R = 70(9.8 - 0.8)$

Answer: (a) 0.8 m s^{-2} downwards (b) 630 N

4 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small x (in radians), use the small angle approximations to find the value that $\frac{\sin 4x \tan 2x}{1 - \cos 2x}$ approaches as $x \to 0$.

$\sin 4x \approx 4x$, $\tan 2x \approx 2x$, $1 - \cos 2x \approx \frac{1}{2}(2x)^2$

Expression $\approx \frac{4x \times 2x}{\frac{4}{2}x^2} = \frac{8x^2}{\frac{4}{2}x^2} = 4$

Answer: 4

5 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(10, −10) and B(−2, 6) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(14, −2) lies on the circle.

Centre = midpoint of AB = (4, −2)
$r^2 = (6)^2 + (-8)^2 = 100$
$(x - 4)^2 + (y + 2)^2 = 100$: $x^2 + y^2 - 8x + 4y - 80 = 0$
(b) $CM^2 = (10)^2 + (0)^2 = 100 = r^2$

Answer: (a) $x^2 + y^2 - 8x + 4y - 80 = 0$

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6 Parametric differentiation: tangent and normal (All boards)

A curve has parametric equations $x = \frac{1}{t}$, $y = t^2$.

Find $\frac{dy}{dx}$ in terms of t, and the equation of the normal at the point where $t = -\frac{1}{2}$, in the form ax + by + c = 0 (integers).

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = -2t^3$

$t = -\frac{1}{2}$: point $(-2, \frac{1}{4})$, gradient $\frac{1}{4}$

Normal gradient $= -1 \div \frac{1}{4} = -4$, so normal: $16x + 4y + 31 = 0$

Answer: $\frac{dy}{dx} = -2t^3$; $16x + 4y + 31 = 0$

7 Conditional normal probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(52, 6^2)$. Find $P(X > 59 \mid X > 52)$ to 3 significant figures.

Since $X > 59$ implies $X > 52$: $P = \frac{P(X > 59)}{P(X > 52)}$

$P(X > 59) = P(Z > 1.17) = 0.122$

$P(X > 52) = P(Z > 0) = 0.500$

$P = \frac{0.122}{0.500} = 0.243$

Answer: 0.243

8 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $-x^2 + 4xy - x - 3y = -51$.

Show that P(−3, 3) lies on the curve and find the equation of the tangent at P in the form ax + by + c = 0 with integer a, b, c.

Differentiate (product rule on the xy term): $(-2x + 4y - 1) + (4x - 3)\frac{dy}{dx} = 0$

At P(−3, 3): $17 + (-15)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = \frac{17}{15}$

Tangent: $y - 3 = \frac{17}{15}(x + 3)$, i.e. $17x - 15y + 96 = 0$

Answer: $17x - 15y + 96 = 0$

9 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Parts are defective with probability 0.2, independently. A box of 7 parts is rejected if it has at least 3 defective. From 6 such boxes, find the probability that exactly 4 are rejected (3 s.f.).

$Y \sim B(7, 0.2)$. $P(\text{rejected}) = P(Y \geq 3) = 1 - 0.852 = 0.148$

$W \sim B(6, 0.148)$ is the number rejected

$P(W = 4) = C(6, 4)(0.148)^4(1 - 0.148)^2 = 0.00523$

Answer: 0.00523

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10 Periodic sequences (All boards)

A sequence is defined by $u_1 = \frac{1}{2}$ and $u_{n+1} = 1 - \frac{1}{u_n}$.

(a) Find u_2 and u_3.

(b) Find u_{200} and $\sum_{r=1}^{200} u_r$.

$u_2 = -1$, $u_3 = 2$, $u_4 = \frac{1}{2}$

$u_4 = u_1$, so the sequence repeats with period 3.

$200 = 3 \times 66 + 2$, so $u_{200} = u_2 = -1$

Sum of one period = $\frac{3}{2}$; 66 full periods plus 2 extra term(s)

Answer: (a) -1, 2 (b) $u_{200} = -1$, sum = $\frac{197}{2}$