Day 117 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 765 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(100 + 70) × V
765 = 85 V, so V = 9
(b) Deceleration = V ÷ 15 = 9 ÷ 15
Answer: (a) V = 9 (b) 0.6 m s^{-2}

2 Equations quadratic in disguise (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 8\sqrt{x} + 15 = 0$.

Let $u = \sqrt{x}$: $u^2 - 8u + 15 = 0$
$(u - 3)(u - 5) = 0$ so $u = 3$ or 5
$\sqrt{x} = 3$ gives $x = 9$; $\sqrt{x} = 5$ gives $x = 25$
Answer: x = 9 or x = 25

3 Symmetric interval about the mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the height (cm), is N(171, 7^2).
Find the symmetric limits about the mean between which the central 90% of values lie (3 s.f.).

Each tail has area 0.05, so P(Z < z) = 0.95
z = 1.64
Limits: 171 ± 1.64 × 7
= 159 and 183
Answer: 159 to 183

4 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{2}} (2\sin 2x + 3\cos x)\,dx$.

Integrate: $[-\cos 2x + 3\sin x]$
Substitute the limits and subtract: 5
Answer: 5

5 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(5 + 2x)^{-1}$ in ascending powers of x, up to and including x^2.
State the range of x for which the expansion is valid.

$(5 + 2x)^{-1} = 5^{-1}(1 + \frac{2x}{5})^{-1}$

$5^{-1} = \frac{1}{5}$

Expand $(1 + y)^{-1}$ with $y = \frac{2x}{5}$

Valid for $|\frac{2x}{5}| < 1$, i.e. $|x| < \frac{5}{2}$

Answer: $\frac{1}{5} - \frac{2}{25}x + \frac{4}{125}x^2$; $|x| < \frac{5}{2}$

6 Proving trigonometric identities (All boards)

Prove the identity
$\sec^2\theta + \operatorname{cosec}^2\theta \equiv 4\operatorname{cosec}^2 2\theta$

$\text{LHS} = \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta} = \frac{1}{\sin^2\theta\cos^2\theta}$

$= \frac{4}{(2\sin\theta\cos\theta)^2} = \frac{4}{\sin^2 2\theta} = \text{RHS}$

Answer: Proved

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7 Inverse of a quadratic function (All boards)

$f(x) = 3x^2 + 12x + 16$, $x \geq -2$.
Find $f^{-1}(x)$ and state its domain.

$f(x) = 3(x + 2)^2 + 4$

Let $y = f(x)$: $(x + 2)^2 = \frac{y - 4}{3}$

$x \geq -2$, so take the positive root: $x = -2 + \sqrt{\frac{y - 4}{3}}$

Answer: $f^{-1}(x) = -2 + \sqrt{\frac{x - 4}{3}}$, $x \geq 4$

8 Car and trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 1200 kg car tows a 300 kg trailer by a light tow-bar on a horizontal road. Driving force 1450 N; resistances 600 N (car), 100 N (trailer). Find (a) the acceleration and the tow-bar tension. At 20 m s^{-1} the tow-bar breaks. (b) How far does the trailer then travel?

Whole system: $1450 - 600 - 100 = 1500\,a$, so $a = 0.5$
Trailer: $T - 100 = 300\,a$, so $T = 100 + 300 \times 0.5 = 250$

(b) After the break the trailer decelerates at $100 \div 300 = \frac{1}{3}$ m s^{-2}

$0 = 20^2 - 2 \times \frac{1}{3} \times s$, so $s = 600$

Answer: (a) 0.5 m s^{-2}, 250 N (b) 600 m

9 Differential equations in context (All boards)

Water drains from a tank so that the depth h metres falls at a rate proportional to $\sqrt{h}$. Initially $h = 64$, and after 6 hours $h = 25$.
Form and solve a differential equation to find how long, in total, the tank takes to empty.

$\frac{dh}{dt} = -k\sqrt{h}$: $\int h^{-\frac{1}{2}}\,dh = \int -k\,dt$, so $2\sqrt{h} = -kt + C$

$t = 0$: $C = 2 \times 8 = 16$, so $2\sqrt{h} = 16 - kt$
$t = 6$, $h = 25$: $10 = 16 - 6k$, so $k = 1$

Empty when $h = 0$: $kt = 16$, so $t = \frac{16}{1} = 16$ hours

Answer: 16 hours

10 Interpreting a regression line (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows data and the regression line $y = 50 - 1.2x$.
(a) Interpret the gradient.
(b) Estimate y when $x = 19$ and comment.

(a) On average, y decreases by 1.2 £000 for each extra 1 years of x
(b) $y = 50 - 1.2 \times 19 = 27.2$
$x = 19$ is outside the range of the data (5 to 14): extrapolation, so unreliable
Answer: (a) y decreases by 1.2 £000 per years (b) 27.2, unreliable