Day 116 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Differentiating sec, cosec, tan and cot (All boards)

y = x cot(3x).

Find the exact gradient at x = $\frac{\pi}{9}$.

Product rule: $\frac{dy}{dx} = \cot(3x) + x(-3\operatorname{cosec}^2(3x))$

At x = $\frac{\pi}{9}$: $\cot(3x) = \frac{\sqrt{3}}{3}$ and the derivative term = -4

Gradient = $\frac{\sqrt{3}}{3} + \frac{\pi}{9} \times (-4) = \frac{3\sqrt{3} - 4\pi}{9}$

Answer: gradient $\frac{3\sqrt{3} - 4\pi}{9}$

2 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A company employs 432 people: 108 Sales, 168 Admin and 156 Production.
A stratified sample of size 36 is taken.
Find the number of people sampled from each group.

Sampling fraction = $\frac{36}{432} = \frac{1}{12}$

Sales: $108 \times \frac{1}{12} = 9$

Admin: $168 \times \frac{1}{12} = 14$

Production: $156 \times \frac{1}{12} = 13$

Answer: Sales 9, Admin 14, Production 13

3 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 5 kg particle hangs in equilibrium from two light strings, as shown. Find the tension in each string (T_1 in the string at angle α). Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = 0.8$, $\cos\alpha = 0.6$, $\sin\beta = \frac{5}{13}$, $\cos\beta = \frac{12}{13}$

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_2 = 0.65\, T_1$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 5\,g = 49$

$T_1(0.8 + 0.25) = 49$

$T_1 = 46.7$, $T_2 = 30.3$

Answer: T_1 = 46.7 N, T_2 = 30.3 N

4 Combined inequalities (All boards)

(a) Solve $3x + 5 \leq 7$.

(b) Solve $x^2 + 7x + 10 > 0$.

(c) Hence find the values of x satisfying both inequalities.

(a) $3x + 5 \leq 7$ so $x \leq \frac{2}{3}$

(b) $(x + 5)(x + 2) > 0$: critical values -5 and -2

(b) gives $x < -5$ or $x > -2$

(c) Overlap of the two sets.

Answer: (a) $x \leq \frac{2}{3}$ (b) $x < -5$ or $x > -2$ (c) $x < -5$ or $-2 < x \leq \frac{2}{3}$

5 Pulled at an angle on rough ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg box is pulled along rough horizontal ground, $\mu = 0.25$, by a rope at angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$.

The tension is 20 N. Find (a) the normal reaction, (b) the acceleration of the box. Take $g = 9.8$.

$\sin\theta = 0.8$, $\cos\theta = 0.6$

(a) Vertical: $R + 20\sin\theta = 4g$, so $R = 23.2$

(b) Horizontal: $20\cos\theta - \mu R = 4a$

$12 - 0.25\times23.2 = 4a$

$a = 1.55$

Answer: (a) 23.2 N (b) 1.55 m s^{-2}

6 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against ln x for $y = Ax^n$.

Find the value of n and the exact value of A.

$\ln y = \ln A + n \ln x$: gradient n, intercept $\ln A$

$n = \frac{8-5}{2-1} = 3$

At (1, 5): $\ln A = 5 - 3 \times 1 = 2$, so $A = e^2$

Answer: n = 3, A = e^2

7 Asymptotes after transformation (All boards)

The curve $y = f(x)$, where $f(x) = \frac{2}{x-4} + 4$, is translated by $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of $y = f(x)$: $x = 4$, $y = 4$

Translate by $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$: $x = 7$, $y = 1$

New curve: $y = \frac{2}{x-7} + 1$

$y = 0$: $\frac{2}{x-7} = -1$, so $x - 7 = -2$

Answer: x = 7, y = 1; crosses x-axis at (5, 0)

8 Combining two data sets (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The marks of two groups: class A has n = 21, mean 57, sd 2;
class B has n = 21, mean 63, sd 2.
Find the mean and standard deviation of all 42 values (3 s.f. for the sd).

$\Sigma x = 21 \times 57 + 21 \times 63 = 2520$, mean = 60

Σx^2 (A) $= 21(2^2 + 57^2) = 68313$

Σx^2 (B) $= 21(2^2 + 63^2) = 83433$

Variance $= \frac{151746}{42} - 60^2 = 13$, so sd = 3.61

Answer: mean 60, sd 3.61

Day 116 **A Level Maths: A Tier 10-a-day: Model Solutions**

9 Optimisation problems (All boards)

A closed cylinder has radius r cm and volume 16π cm^3.

Show that its surface area is $A = 2\pi r^2 + \frac{32\pi}{r}$, and find the minimum value of A in terms of π.

$h = \frac{16}{r^2}$, so $A = 2\pi r^2 + 2\pi rh = 2\pi r^2 + \frac{32\pi}{r}$

$\frac{dA}{dr} = 4\pi r - \frac{32\pi}{r^2} = 0$, so $r^3 = 8$ and $r = 2$

$\frac{d^2A}{dr^2} = 4\pi + \frac{64\pi}{r^3} > 0$, so a minimum

$A = 2\pi \times 4 + \frac{32\pi}{2} = 24\pi$

Answer: A_min = 24π cm^2 (r = 2)

10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{-3x - 5}{(1 - 3x)(1 + 3x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - 3x)} + \frac{B}{(1 + 3x)}$: $A = -3$, $B = -2$

(b) $-3(1 - 3x)^{-1} = -3(1 + 3x + 9x^2 ...)$

Add the series for both fractions.

(c) Need $|3x| < 1$ and $|3x| < 1$

Answer: (a) $\frac{-3}{(1 - 3x)} + \frac{-2}{(1 + 3x)}$ (b) $-5 - 3x - 45x^2$ (c) $|x| < \frac{1}{3}$