Day 114 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Discrete random variable with unknown constant (Edexcel, OCR, MEI, Eduqas, CIE)

The random variable X has P(X = x) = k(7 − x) for x = 2, 3, 4, 5, 6.

(a) Show that k = $\frac{1}{15}$. (b) Find E(X) and Var(X).

(c) Find P(X ≥ 3).

(a) Σ P(X = x) = 1: 5 + 4 + 3 + 2 + 1 = 15, so 15k = 1

(b) E(X) = $2 \times \frac{1}{3} + 3 \times \frac{4}{15} + 4 \times \frac{1}{5} + 5 \times \frac{2}{15} + 6 \times \frac{1}{15} = \frac{10}{3}$

$E(X^2) = \frac{38}{3}$, so $Var(X) = \frac{38}{3} - (\frac{10}{3})^2 = \frac{14}{9}$

(c) $P(X \geq 3) = \frac{4}{15} + \frac{1}{5} + \frac{2}{15} + \frac{1}{15} = \frac{2}{3}$

Answer: (b) E(X) = $\frac{10}{3}$, Var(X) = $\frac{14}{9}$ (c) $\frac{2}{3}$

2 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 10 kg is pulled at constant speed along rough horizontal ground, μ = 0.2, by a force of P N at angle θ above the horizontal, tan θ = $\frac{4}{3}$. Find P and the normal reaction. Take g = 9.8. Give answers to 3 s.f. if not exact.

Constant speed, so the resultant force is zero. sin θ = 0.8, cos θ = 0.6
Vertical: R = 10 g − P sin θ
Horizontal: P cos θ = μR = 0.2(10 g − P sin θ)
P(0.6 + 0.16) = 0.2×10 g
P = 25.8, R = 77.4
Answer: P = 25.8 N, R = 77.4 N

3 Index equations solved with logarithms (All boards)

Solve $7^{3x-2} = 5^{3x-2}$.
Give your answer to 3 significant figures.

Take logs: (3x − 2) ln 7 = (3x − 2) ln 5
Collect x terms: x(3 ln 7 − 3 ln 5) = −2 ln 5 + 2 ln 7
$x = \frac{-2 \ln 5 + 2 \ln 7}{3 \ln 7 - 3 \ln 5}$
x = 0.667
Answer: x = 0.667

4 Venn diagrams and conditional probability (All boards)

P(A) = 0.35, P(B) = 0.4 and P(A ∩ B) = 0.1.
Find
(a) P(A ∪ B)
(b) P(A | B′)
(c) P(B′ | A′)

(a) P(A ∪ B) = P(A) + P(B) − P(A ∩ B) = 0.35 + 0.4 − 0.1 = 0.65

(b) $P(A \mid B') = \frac{P(A \cap B')}{P(B')} = \frac{0.35 - 0.1}{1 - 0.4} = \frac{5}{12}$

(c) $P(B' \mid A') = \frac{P(A' \cap B')}{P(A')} = \frac{1 - 0.65}{1 - 0.35} = \frac{7}{13}$

Answer: (a) 0.65 (b) $\frac{5}{12}$ (c) $\frac{7}{13}$

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5 Line meets curve: length of chord (All boards)

The line $y = -2x - 3$ meets the curve $y = x^2 + 6x + 12$ at A and B.
(a) Find the coordinates of A and B.
(b) Find the exact length AB.

$x^2 + 6x + 12 = -2x - 3$ gives $x^2 + 8x + 15 = 0$
$(x + 5)(x + 3) = 0$: $x = -5$ or -3
A$(-5, 7)$, B$(-3, 3)$
$AB^2 = 2^2 + (-4)^2 = 20$
Answer: A$(-5, 7)$, B$(-3, 3)$; AB = $2\sqrt{5}$

6 Exact definite integrals of standard functions (All boards)

Find the exact value of
$\int_1^2 (\frac{3}{x} + \frac{7}{x^2})\,dx$.

Integrate: $[3 \ln x - \frac{7}{x}]$

Substitute the limits and subtract: $\frac{6 \ln 2 + 7}{2}$

Answer: $\frac{6 \ln 2 + 7}{2}$

7 Smooth incline held by a string (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 2 kg particle rests on a smooth plane inclined at α, $\tan \alpha = \frac{5}{12}$, held by a string in the vertical plane through a line of greatest slope, at angle θ above the slope, $\tan \theta = \frac{5}{12}$. Find the tension and the normal reaction. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin \alpha = \frac{5}{13}$, $\cos \alpha = \frac{12}{13}$, $\sin \theta = \frac{5}{13}$, $\cos \theta = \frac{12}{13}$
Along the plane: $T \cos \theta = 2g \sin \alpha$, so $T = 8.17$
Perpendicular: $R + T \sin \theta = 2g \cos \alpha$
$R = 2g \cos \alpha - T \sin \theta = 18.1 - 8.17 \times 0.385$
Answer: T = 8.17 N, R = 15.0 N

8 Exponential equations: quadratic in b^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3 \times 3^{2x} - 28 \times 3^x + 9 = 0$.

Let $y = 3^x$. Then $3y^2 - 28y + 9 = 0$

$3(y - \frac{1}{3})(y - 9) = 0$

$y = \frac{1}{3}$ or $y = 9$

$3^x = \frac{1}{3}$ gives $x = -1$; $3^x = 9$ gives $x = 2$

Answer: x = −1 or x = 2

9 Binomial expansion: product with a bracket (All boards)

Find the coefficient of x^4 in the expansion of
$(3 + 3x)(2 - x)^6$.

In $(2 - x)^6$: coefficient of x^3 is -160, of x^4 is 60
Coefficient $= 3 \times 60 + 3 \times (-160)$
Answer: −300

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10 Integration by parts (All boards)

Use integration by parts to find the exact value of

$\int_0^{\frac{\pi}{6}} x \cos 3x \, dx$.

$u = x, \frac{dv}{dx} = \cos 3x: \int x \cos 3x \, dx = [\frac{x \sin 3x}{3}] - \int \frac{\sin 3x}{3} \, dx$

$= [\frac{x \sin 3x}{3} + \frac{\cos 3x}{9}]$

$= \frac{\pi - 2}{18}$

Answer: $\boldsymbol{\frac{\pi - 2}{18}}$