Day 113 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (1, 2, −1), (2, 3, −3) and (2, 5, 5).
Find angle ABC, in degrees to 1 d.p.

BA = $\begin{pmatrix}-1\\-1\\2\end{pmatrix}$, BC = $\begin{pmatrix}0\\2\\8\end{pmatrix}$

BA·BC = 14

|BA| = $\sqrt{6}$, |BC| = $\sqrt{68}$

cos ABC = $\frac{14}{\sqrt{6}\times\sqrt{68}}$ = 0.6931, so angle ABC = 46.1°

Answer: 46.1°

2 Inverse of a quadratic function (All boards)

$f(x) = 3x^2 + 18x + 29$, $x \geq -3$.
Find $f^{-1}(x)$ and state its domain.

$f(x) = 3(x + 3)^2 + 2$

Let $y = f(x)$: $(x + 3)^2 = \frac{y - 2}{3}$

$x \geq -3$, so take the positive root: $x = -3 + \sqrt{\frac{y - 2}{3}}$

Answer: $f^{-1}(x) = -3 + \sqrt{\frac{x - 2}{3}}$, $x \geq 2$

3 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

For a sample of 13 pairs of values of daily rainfall and ice cream sales, r = 0.337.
The critical value for a one-tailed test at the 5% level is 0.4762.
Test, at this level, whether there is positive correlation between the two variables.

H_0: ρ = 0, H_1: ρ > 0

Critical value 0.4762

r = 0.337 < 0.4762

This is not in the critical region, so do not reject H_0.

There is insufficient evidence of correlation.

Answer: Do not reject H_0: insufficient evidence

4 Collinear points and ratios (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A, B and C have position vectors $\begin{pmatrix}0\\-2\\3\end{pmatrix}$, $\begin{pmatrix}3\\0\\0\end{pmatrix}$ and $\begin{pmatrix}k\\-4\\6\end{pmatrix}$

relative to O, where k is a constant. Given that A, B and C are collinear, find k and the ratio AB : BC.

AB = $\begin{pmatrix}3\\2\\-3\end{pmatrix}$, and AC = $\begin{pmatrix}k\\-2\\3\end{pmatrix}$

AC = λAB: component 1 gives −3 = 3 λ, so λ = −1

Hence k = 0 + (−1) × 3 = −3

AC = −1 AB, so BC = −2 AB: AB : BC = 1 : 2

Answer: k = −3, AB : BC = 1 : 2

5 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers m and n such that 10m + 15n = 67.

Assume 10m + 15n = 67 for integers m, n.

Then 5(2m + 3n) = 67, so 5 divides 67.

But 67 is not a multiple of 5. Contradiction.

Answer: no such integers exist

6 Horizontal projection (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 24 m s^{-1} from a point 4.9 m above horizontal ground. Find (a) the horizontal distance to the landing point, (b) the speed and direction (angle below the horizontal, 1 d.p.) on landing. Take g = 9.8. Give answers to 3 s.f. if not exact.

Vertical: $4.9 = \frac{1}{2} \times 9.8 \times t^2$, so t = 1
(a) Horizontal distance = 24 × 1
(b) Vertical speed on landing = 9.8×1 = 9.8; horizontal speed = 24
Speed = $\sqrt{(24^2 + 9.8^2)} = 25.9$
$\tan \theta = 9.8 \div 24$, $\theta = 22.2°$

Answer: (a) 24 m (b) 25.9 m s^{-1} at 22.2° below the horizontal

7 Correcting a recorded value (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

12 values have $\Sigma x = 636$ and $\Sigma x^2 = 34008$.
One value was recorded as 39 but should have been 58.
Find the correct mean and standard deviation (3 s.f.).

$\Sigma x = 636 - 39 + 58 = 655$
$\Sigma x^2 = 34008 - 39^2 + 58^2 = 35851$
Mean = $\frac{655}{12} = 54.6$
sd = $\sqrt{\frac{35851}{12} - (54.6)^2} = 2.87$

Answer: mean 54.6, sd 2.87

8 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $12x + 5y = 128$ meets the circle $(x - 4)^2 + (y - 3)^2 = 36$ at the points A and B. Find the exact length of AB.

Centre (4, 3), radius 6. Distance from centre to line:
$d = \frac{|-65|}{13} = 5$
Half-chord$^2 = 36 - (5)^2 = 11$
AB = $2\sqrt{11}$

Answer: $2\sqrt{11}$

9 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against x for $y = A e^{kx}$.
Find the exact values of A and k.

ln y = ln A + kx, a straight line with gradient k and intercept ln A
Intercept ln A = 1, so $A = e^1$
Gradient $k = \frac{7 - 1}{4 - 0} = \frac{3}{2}$

Answer: $A = e^1$, $k = \frac{3}{2}$

10 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 33 m s^{-1} at angle α above the horizontal, $\tan \alpha = \frac{24}{7}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity (9.24, 31.68). At time T the velocity is (9.24, 31.68 − 9.8T).
Perpendicular: scalar product = 0: $(9.24)^2 + 31.68(31.68 - 9.8T) = 0$
$33^2 - 9.8T \times 31.68 = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 33 \div (9.8 \times 0.96) = \frac{1375}{392}$

(b) $h = 31.68 \times \frac{1375}{392} - 4.9 \times \left(\frac{1375}{392}\right)^2 = \frac{318835}{6272}$

Answer: (a) T = 3.51 s (b) 50.8 m