Day 112 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Finding an unknown standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is normally distributed with mean 69 and standard deviation σ.
Given that P(X > 70) = 0.3085, find σ to 3 significant figures.

P(Z > z) = 0.3085, so z = 0.500

$\frac{70 - 69}{\sigma} = 0.500$

$\sigma = \frac{1}{0.500} = 2.00$

Answer: σ = 2.00

2 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 - 10x - 2y - 19 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P(5, −4) lies inside or outside the circle.

Complete the square: $(x - 5)^2 + (y - 1)^2 = 45$
Centre (5, 1), radius $3\sqrt{5}$
Distance^2 from centre to P = $0^2 + (-5)^2 = 25$
Compare with $r^2 = 45$

Answer: (a) centre (5, 1), radius $3\sqrt{5}$ (b) P is inside the circle

3 Conservation of momentum (CIE)

Particles A (8 kg) and B (2 kg) move in the same direction on a smooth horizontal line at 8 m s^{-1} and 2 m s^{-1}. They collide and coalesce. Find (a) the speed of the combined particle, (b) the kinetic energy lost in the collision.

Conservation of momentum (A's direction positive):
8×8 + 2×2 = 10 v, so v = 6.8
KE before = ½×8×8^2 + ½×2×2^2 = 260
KE after = ½×10×6.8^2 = 231.2
Loss = 260 − 231.2

Answer: (a) 6.8 m s^{-1} (b) 28.8 J

4 Poisson probabilities (CIE)

Cars pass a junction at a mean rate of 3 per minute, as a Poisson process.
Find the probability that in one such period there are
(a) exactly 3 (b) at least 3 (c) fewer than 3 (3 s.f.).

X ~ Po(3)
(a) $P(X = 3) = e^{-3}\, 3^3/6! = 0.224$
(b) $1 - P(X \leq 2) = 1 - 0.423 = 0.577$
(c) $P(X \leq 2) = 0.423$

Answer: (a) 0.224 (b) 0.577 (c) 0.423

5 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 + 4x - 8$.
Use the Newton-Raphson method with $x_0 = 1.5$ to find x_1 and x_2, giving your answers to 4 d.p.

$f'(x) = 3x^2 + 4$

$x_1 = 1.5 - \frac{f(1.5)}{f'(1.5)} = 1.5 - \frac{1.3750}{10.7500} = 1.3721$

$x_2 = 1.3721 - \frac{f(1.3721)}{f'(1.3721)} = 1.3647$

Answer: $x_1 = 1.3721$, $x_2 = 1.3647$

6 Acceleration function of time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on the x-axis. At time t s its acceleration is $(6t - 6)$ m s^{-2}. When t = 0 it is at O with velocity -2 m s^{-1}. Find (a) its velocity when the acceleration is zero, (b) its displacement from O then.

a = 0 when t = 1

$v = \int a\,dt = 3t^2 - 6t - 2$ (using v = −2 when t = 0)

v(1) = −5

$x = \int v\,dt = t^3 - 3t^2 - 2t$ (x = 0 when t = 0)

x(1) = −4

Answer: (a) -5 m s^{-1} (b) -4 m

7 Partial fractions and integration (All boards)

(a) Express $\frac{-6x - 33}{(x + 5)(x + 6)}$ in partial fractions.

(b) Hence find $\int_0^4 \frac{-6x - 33}{(x + 5)(x + 6)}\,dx$, giving the answer in the form ln k.

$-6x - 33 \equiv A(x + 6) + B(x + 5)$: x = −6 gives A = −3; x = −5 gives B = −3

$I = [-3\ln(x + 5) - 3\ln(x + 6)]_0^4$

$= -3\ln\frac{9}{5} - 3\ln\frac{10}{6}$

$= \ln\frac{1}{27}$

Answer: (a) $\frac{-3}{x + 5} - \frac{3}{x + 6}$; (b) $\ln\frac{1}{27}$

8 Asymptotes after transformation (All boards)

The curve y = f(x), where $f(x) = \frac{2}{x - 2} + 4$, is translated by $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of y = f(x): x = 2, y = 4

Translate by $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$: x = 0, y = 5

New curve: $y = \frac{2}{x} + 5$

y = 0: $x = -\frac{2}{5}$

Answer: x = 0, y = 5; crosses x-axis at $(-\frac{2}{5}, 0)$

9 Equations with sec x and tan x (All boards)

Solve $\sec^2 x - 5\tan x + 5 = 0$
for $0° \le x \le 360°$. Give answers to 1 d.p.

Use $\sec^2 x \equiv 1 + \tan^2 x$: $\tan^2 x - 5\tan x + 6 = 0$

tan x = 2 or tan x = 3

tan x = 2: x = 63.4°, 243.4°

tan x = 3: x = 71.6°, 251.6°

Answer: x = 63.4°, 71.6°, 243.4°, 251.6°

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10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{4x - 2}{(1 - x)(1 + x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - x)} + \frac{B}{(1 + x)}$: A = 1, B = −3

(b) $1(1 - x)^{-1} = 1(1 + x + 1x^2 ...)$

Add the series for both fractions.

(c) Need |x| < 1 and |x| < 1

Answer: (a) $\frac{1}{(1 - x)} + \frac{-3}{(1 + x)}$ **(b)** $-2 + 4x - 2x^2$ **(c)** $|x| < 1$