Day 111 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix}1\\1\end{pmatrix}$ and $\begin{pmatrix}-5\\-2\end{pmatrix}$. The point P lies on AB with AP : PB = 1 : 2.

Find the position vector of P and the exact distance OP.

$\text{OP} = \text{OA} + \frac{1}{3}\text{AB} = \begin{pmatrix}1\\1\end{pmatrix} + \frac{1}{3}\begin{pmatrix}-6\\-3\end{pmatrix}$

$= \begin{pmatrix}-1\\0\end{pmatrix}$

$\text{OP} = \sqrt{(-1)^2 + 0^2} = \sqrt{1} = 1$

Answer: $\begin{pmatrix}\mathbf{-1}\\\mathbf{0}\end{pmatrix}$**, OP = 1**

2 Transformations of a turning point (All boards)

The curve y = f(x) has a maximum point at (2, 3).

Find the coordinates of the corresponding point on:

(a) y = −f(x)

(b) y = f(x) − 5

(c) y = f(−x)

(a) y = −f(x): (2, −3)

(b) y = f(x) − 5: (2, −2)

(c) y = f(−x): (−2, 3)

A stretch or translation moves the point in the stated direction only.

Answer: (a) (2, −3); (b) (2, −2); (c) (−2, 3)

3 Binomial mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X has a binomial distribution with mean 4 and variance 3.

(a) Find n and p.

(b) Find P(X = 3) to 3 significant figures.

(a) np = 4 and npq = 3

$q = \frac{npq}{np} = \frac{3}{4} = 0.75$, so p = 0.25

$n = \frac{4}{0.25} = 16$

(b) $P(X = 3) = C(16, 3)(0.25)^3(0.75)^{13} = 0.208$

Answer: (a) n = 16, p = 0.25 (b) 0.208

4 Horizontal projection (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 15 m s^{-1} from a point 4.9 m above horizontal ground. Find (a) the horizontal distance to the landing point, (b) the speed and direction (angle below the horizontal, 1 d.p.) on landing. Take g = 9.8. Give answers to 3 s.f. if not exact.

Vertical: $4.9 = \frac{1}{2}\times 9.8\times t^2$, so t = 1

(a) Horizontal distance = 15 × 1

(b) Vertical speed on landing = 9.8×1 = 9.8; horizontal speed = 15

Speed = $\sqrt{(15^2 + 9.8^2)} = 17.9$

$\tan\theta = 9.8 \div 15$, $\theta = 33.2°$

Answer: (a) 15 m (b) 17.9 m s^{-1} at 33.2° below the horizontal

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5 Sum of multiples in a range (All boards)

Find the sum of all the integers from 1 to 87 inclusive that are not multiples of 13.

Sum of 1 to 87 = $\frac{87 \times 88}{2} = 3828$

Multiples of 13 up to 87: 13, 26, ..., 78 (6 terms)

Their sum = $\frac{6}{2}(13 + 78) = 273$

Required sum = $3828 - 273$

Answer: 3555

6 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

Water pours into a cone, vertex down, at 20 cm^3s^{-1}. The surface radius is always $\frac{h}{2}$.

Find the rate at which the depth is rising when h = 5 cm.

$V = \frac{1}{3}\pi(h/2)^2h = \frac{\pi h^3}{12}$, so $\frac{dV}{dh} = \frac{\pi h^2}{4}$

$\frac{dh}{dt} = \frac{dV}{dt \div dV/dh} = \frac{20 \times 4}{\pi h^2}$

h = 5: $\frac{dh}{dt} = \frac{16}{5\pi}$ cm s^{-1}

Answer: $\frac{16}{5\pi}$ cm/s

7 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is -8 and the coefficient of x^2 is 28. Find n and k.

$nk = -8$ and $\frac{n(n-1)}{2}k^2 = 28$

$(n-1)k = \frac{2 \times 28}{-8} = -7$ (dividing the second equation by the first)

$n\,k - (n-1)\,k = k = -8 - (-7) = -1$

$n = \frac{-8}{-1} = 8$

Answer: n = 8, k = −1

8 Equations with sec x and tan x (All boards)

Solve $2\sec^2\theta - 6\tan\theta + 2 = 0$
for $-180° \le \theta \le 180°$. Give answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $2\tan^2\theta - 6\tan\theta + 4 = 0$
$\tan\theta = 2$ or $\tan\theta = 1$
$\tan\theta = 2$: $\theta = -116.6°, 63.4°$
$\tan\theta = 1$: $\theta = -135.0°, 45.0°$
Answer: θ = −135.0°, −116.6°, 45.0°, 63.4°

9 Range of horizontal force on a rough slope (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 3 kg particle is in equilibrium on a rough plane inclined at α, $\tan\alpha = \frac{4}{3}$, $\mu = 0.2$. A horizontal force of P N, in the vertical plane through a line of greatest slope, pushes it towards the plane. Find the least and greatest values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = 0.8$, $\cos\alpha = 0.6$, weight W = 29.4 N. Normal reaction $R = W\cos\alpha + P\sin\alpha$
Least P: about to slide down, friction $F = \mu R$ up the slope
$P\cos\alpha + \mu(W\cos\alpha + P\sin\alpha) = W\sin\alpha \rightarrow P = W(\sin\alpha - \mu\cos\alpha) \div (\cos\alpha + \mu\sin\alpha) = 26.3$
Greatest P: about to slide up, friction down the slope
$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha) \rightarrow P = W(\sin\alpha + \mu\cos\alpha) \div (\cos\alpha - \mu\sin\alpha) = 61.5$
Answer: least 26.3 N, greatest 61.5 N

10 Conditional binomial probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(20, 0.4). Find P(X = 6 | X ≥ 4) to 3 significant figures.

$P(X = 6 \mid X \geq 4) = \frac{P(X = 6)}{P(X \geq 4)}$ (since X = 6 implies X ≥ 4)

P(X = 6) = 0.124

P(X ≥ 4) = 1 − P(X ≤ 3) = 0.984

Ratio = 0.126

Answer: 0.126