Day 110 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Binomial probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(11, 0.7). Find, to 3 significant figures:
(a) P(X = 10) (b) P(X ≥ 10) (c) P(X ≤ 9)

(a) $C(11, 10)(0.7)^{10}(0.3)^{1} = 0.0932$
(b) $1 - P(X \leq 9) = 1 - 0.887 = 0.113$
(c) $P(X \leq 9) = 0.887$
Answer: (a) 0.0932 (b) 0.113 (c) 0.887

2 Arc length and sector area (All boards)

A sector of a circle has area 38.4 cm^2 and angle 1.2 radians.
Find the radius, the arc length and the perimeter of the sector.

Area $= \frac{1}{2}r^2\theta$: $38.4 = \frac{1}{2}r^2 \times 1.2$, so $r^2 = 64 = 8^2$

r = 8 cm; arc length $s = r\theta = 8 \times 1.2 = 9.6$ cm
Perimeter $= 2r + s = 16 + 9.6 = 25.6$ cm
Answer: r = 8 cm, arc 9.6 cm, perimeter 25.6 cm

3 Binomial mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X has a binomial distribution with mean 14 and variance 10.5.
(a) Find n and p.
(b) Find P(X = 16) to 3 significant figures.

(a) np = 14 and npq = 10.5

$q = \frac{npq}{np} = \frac{10.5}{14} = 0.75$, so p = 0.25

$n = \frac{14}{0.25} = 56$

(b) $P(X = 16) = C(56, 16)(0.25)^{16}(0.75)^{40} = 0.0975$
Answer: (a) n = 56, p = 0.25 (b) 0.0975

4 Line and curve intersection (diagram) (All boards)

The diagram shows the curve $y = x^2 - x - 5$ and the line $y = -x - 4$, meeting at A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the midpoint of AB.

$x^2 - x - 5 = -x - 4$: $x^2 - 1 = 0$
$(x + 1)(x - 1) = 0$, $x = -1$ or 1
A(−1, −3), B(1, −5)
Midpoint: average the coordinates
Answer: (a) A(−1, −3), B(1, −5) (b) (0, −4)

5 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value of $\sin(\arcsin \frac{3}{5} + \arcsin \frac{7}{25})$.

Let $A = \arcsin \frac{3}{5}$ and $B = \arcsin \frac{7}{25}$, both acute

$\cos A = \frac{4}{5}$, $\cos B = \frac{24}{25}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B = \frac{3}{5} \times \frac{24}{25} + \frac{4}{5} \times \frac{7}{25} = \frac{4}{5}$

Answer: $\frac{4}{5}$

6 Connected particles on a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest and A moves up the rough plane. Find (a) the acceleration, (b) the tension in the light string. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$; friction $F = \mu R = 0.2 \times 4\, g \cos\alpha$

B: $10g - T = 10a$

A: $T - 4g\sin\alpha - F = 4a$

Adding: $g(10 - \frac{20}{13} - \frac{48}{65}) = 14a$

(a) $a = 5.41$; (b) $T = 10(g - a)$

Answer: (a) 5.41 m s^{-2} (b) 43.9 N

7 Tangent through the origin (All boards)

The tangent to the curve $y = e^{2x}$ at the point P passes through the origin. Find the coordinates of P and the equation of the tangent.

$\frac{dy}{dx} = 2e^{2x}$; at $x = a$ the tangent is $y - e^{2a} = 2e^{2a}(x - a)$

Through (0, 0): $-e^{2a} = -2a\, e^{2a}$, so $2a = 1$ and $a = \frac{1}{2}$

P = $(\frac{1}{2}, e)$ and gradient $ke = 2e$

Answer: P$(\frac{1}{2}, e)$; y = 2ex

8 Recurrence relations: limits (All boards)

$u_1 = 0$ and $u_{n+1} = \frac{1}{2}u_n + 3$.

(a) Find the limit L of the sequence as $n \to \infty$.

(b) Find the least n for which $|u_n - L| < 0.1$.

(a) $L = \frac{1}{2}L + 3$ gives $L = \frac{3}{1 - \frac{1}{2}} = 6$

(b) $u_n - L = (u_1 - L)\, r^{n-1} = (-6) \times \frac{1}{2}^{n-1}$

$|-6| \times \frac{1}{2}^{n-1} < \frac{1}{10}$

Check: n = 6 gives 0.18750, n = 7 gives 0.09375

Answer: (a) L = 6 (b) n = 7

9 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{-7x - 1}{(1 - 3x)(1 - x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - 3x)} + \frac{B}{(1 - x)}$: A = −5, B = 4

(b) $-5(1 - 3x)^{-1} = -5(1 + 3x + 9x^2 \ldots)$

Add the series for both fractions.

(c) Need $|3x| < 1$ and $|x| < 1$

Answer: (a) $\frac{-5}{(1 - 3x)} + \frac{4}{(1 - x)}$ (b) $-1 - 11x - 41x^2$ (c) $|x| < \frac{1}{3}$

10 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle α above the horizontal, $\tan\alpha = \frac{4}{3}$, and passes through A, 24 m horizontally from O and 22 m above the ground. Find the speed of projection. Take g = 9.8.

Let the speed be U. Horizontal: $24 = U\cos\alpha \times t$, so $t = 24 \div (0.6\,U)$

Vertical: $22 = U\sin\alpha \times t - 4.9t^2$

Eliminate t: $y = x\tan\alpha - 4.9x^2 \div (U^2\cos^2\alpha)$

$22 = 32 - 4.9\times24^2 \div (0.36\,U^2)$

$U^2 = 7840 \div 10 = 784$, $U = 28$

Answer: 28 m s^{-1}